Im folgenden gilt: $c = 1$

Aufgabe H1

Für jedes Teilchen gilt: $p_i^2 = m_i^2$, der Erhaltungssatz besagt: $p_1 + p_2 = p_3 + p_4$

(a)

$$s + t + u = p_1^2 + p_2^2 + p_3^2 + p_4^2 + 2p_1(p_1 + \underbrace{p_2 - p_3 - p_4}_{-p_1}) \tag{1}$$

$$= \sum m_i^2 \tag{2}$$

(b) Aus der Impulserhaltung folgt als weitere Schreibweise für s: $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$, analog die weiteren Variablen. Damit lassen sich alle Skalarprodukte im folgenden durch die Mandelstammvariablen darstellen (nicht explizit umgestellt):

$$s = m_1^2 + m_2^2 + 2p_1p_2, \qquad s = m_3^2 + m_4^2 + 2p_3p_4 \tag{3}$$

$$t = m_1^2 + m_3^2 - 2p_1p_3, \qquad t = m_2^2 + m_4^2 - 2p_2p_4 \tag{4}$$

$$u = m_1^2 + m_4^2 - 2p_1p_4, \qquad u = m_2^2 + m_3^2 - 2p_2p_3 \tag{5}$$

Aufgabe H2

$$p_\Delta = p_p + p_{\pi^0} \tag{6}$$

$$(m_\Delta, 0, 0, 0) = (\gamma_p m_p, p_1, 0, 0) + (\gamma_{\pi^0} m_{\pi^0}, -p_1, 0, 0) \tag{7}$$

$$\Rightarrow \quad m_\Delta = \sqrt{m_p^2 + p_1^2} + \sqrt{m_{\pi^0}^2 + p_1^2} \tag{8}$$

$$m_\Delta^2 = m_p^2 + p_1^2 + m_{\pi^0}^2 + p_1^2 + 2\sqrt{(m_p^2 + p_1^2)(m_{\pi^0}^2 + p_1^2)} \tag{9}$$

$$m_\Delta^2 - m_p^2 - m_{\pi^0}^2 - 2p_1^2 = 2\sqrt{(m_p^2 + p_1^2)(m_{\pi^0}^2 + p_1^2)} \tag{10}$$

$$(m_\Delta^2 - m_p^2 - m_{\pi^0}^2 - 2p_1^2)^2 = 4(m_p^2 m_{\pi^0}^2 + p_1^2(m_p^2 + m_{\pi^0}^2) + p_1^4) \tag{11}$$

$$m_\Delta^4 - m_\Delta^2(m_p^2 + m_{\pi^0}^2 + 2p_1^2) \tag{12}$$

$$+m_p^4 + m_p^2(-m_\Delta^2 + m_{\pi^0}^2 + 2p_1^2) \tag{13}$$

$$+m_{\pi^0}^4 + m_{\pi^0}^2(-m_\Delta^2 + m_p^2 + 2p_1^2) \tag{14}$$

$$+4p_1^4 + 2p_1^2(-m_\Delta^2 + m_p^2 + m_{\pi^0}^2) = 4(m_p^2 m_{\pi^0}^2 + p_1^2(m_p^2 + m_{\pi^0}^2) + p_1^4) \tag{15}$$

$$m_\Delta^4 + m_p^4 + m_{\pi^0}^4 - 2m_\Delta^2(m_p^2 + m_{\pi^0}^2 + 2p_1^2) = 2m_p^2 m_{\pi^0}^2 \tag{16}$$

$$4m_\Delta^2 p_1^2 = m_\Delta^4 + m_p^4 + m_{\pi^0}^4 - 2m_\Delta^2(m_p^2 + m_{\pi^0}^2) - 2m_p^2 m_{\pi^0}^2 \tag{17}$$

$$\Rightarrow \quad p_1 = \sqrt{\frac{m_\Delta^4 + m_p^4 + m_{\pi^0}^4 - 2m_\Delta^2(m_p^2 + m_{\pi^0}^2) - 2m_p^2 m_{\pi^0}^2}{4m_\Delta^2}} \tag{18}$$

Daraus folgt $p_1 = 229.229\,\mathrm{MeV}$, sowie $E_p = 965.911\,\mathrm{MeV}$ und $E_{\pi^0} = 266.089\,\mathrm{MeV}$.

Aufgabe H3

Nehme als Tensor für den Minkowskiraum $\eta_{\mu\nu} = \mathrm{diag}\{-1, 1, 1, 1\}$

(a)

$$x_+ = \eta_{\mu\nu} x^+ = \frac{1}{\sqrt{2}} \eta_{\mu\nu}(x^0 + x^1) = -x^- \tag{19}$$
$$x_- = \eta_{\mu\nu} x^- = \frac{1}{\sqrt{2}} \eta_{\mu\nu}(x^0 - x^1) = -x^+ \tag{20}$$
$$x_i = x^i \tag{21}$$

(b)

$$x_+ x^+ = -x^- x^+ = \frac{1}{2}(x_0^2 - x_1^2) = x_- x^- \quad \Rightarrow \quad \eta'_{\mu\nu} = \text{diag}\{-1, -1, 1, 1\} \tag{22}$$

(c)

$$\frac{\partial}{\partial x^+} = \left(\frac{\partial x^0}{\partial x^+} \frac{\partial}{\partial x^0} + \frac{\partial x^1}{\partial x^+} \frac{\partial}{\partial x^1} \right) \tag{23}$$
$$\frac{\partial}{\partial x^+} = \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^1} \right) \tag{24}$$
$$\Rightarrow \quad \frac{\partial}{\partial x^-} = \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial x^1} \right) \tag{25}$$

Da aus der Definition folgt: $x^0 = \frac{1}{\sqrt{2}}(x^+ + x^-),\ x^1 = \frac{1}{\sqrt{2}}(x^+ - x^-)$.

(d) In den ursprünglichen Koordinaten sind die Transformationen gegeben durch ($\Phi = \operatorname{artanh} v$):

$$\text{Boost in } x^1: \qquad x^{0'} = \cosh\Phi x^0 - \sinh\Phi x^1 \tag{26}$$
$$x^{1'} = -\sinh\Phi x^0 + \cosh\Phi x^1 \tag{27}$$

$$\text{Boost in } x^2: \qquad x^{0'} = \cosh\Phi x^0 - \sinh\Phi x^2 \tag{28}$$
$$x^{2'} = -\sinh\Phi x^0 + \cosh\Phi x^2 \tag{29}$$
$$\tag{30}$$

Eingesetzt in die Lichtkegelkoordinaten folgt:

$$\text{Boost in } x^1: \qquad x^{+'} = \frac{1}{\sqrt{2}}(x^{0'} + x^{1'}) \tag{31}$$
$$= \frac{1}{\sqrt{2}}(\cosh\Phi - \sinh\Phi)(x^0 + x^1) \tag{32}$$
$$= \underline{(\cosh\Phi - \sinh\Phi)x^+} \tag{33}$$

$$x^{-'} = \frac{1}{\sqrt{2}}(x^{0'} - x^{1'}) \tag{34}$$
$$= \frac{1}{\sqrt{2}}(\cosh\Phi + \sinh\Phi)(x^0 - x^1) \tag{35}$$
$$= \underline{(\cosh\Phi + \sinh\Phi)x^-} \tag{36}$$

sowie

$$\text{Boost in } x^2: \quad x^{+'} = \frac{1}{\sqrt{2}}(x^{0'} + x^{1'}) \tag{37}$$

$$= \frac{1}{\sqrt{2}}(\cosh\Phi x^0 + x^1 - \sinh\Phi x^2) \tag{38}$$

$$= \frac{1}{2}(\cosh\Phi(x^+ + x^-) + (x^+ - x^-) - \sqrt{2}\sinh\Phi x^2) \tag{39}$$

$$= \underline{\frac{\cosh\Phi + 1}{2}x^+ + \frac{\cosh\Phi - 1}{2}x^- - \frac{\sinh\Phi}{\sqrt{2}}x^2} \tag{40}$$

$$x^{-'} = \frac{1}{\sqrt{2}}(x^{0'} - +x^{1'}) \tag{41}$$

$$= \frac{1}{\sqrt{2}}(\cosh\Phi x^0 - x^1 - \sinh\Phi x^2) \tag{42}$$

$$= \frac{1}{2}(\cosh\Phi(x^+ + x^-) - (x^+ - x^-) - \sqrt{2}\sinh\Phi x^2) \tag{43}$$

$$= \underline{\frac{\cosh\Phi - 1}{2}x^+ + \frac{\cosh\Phi + 1}{2}x^- - \frac{\sinh\Phi}{\sqrt{2}}x^2} \tag{44}$$

$$x^{2'} = -\sinh\Phi x^0 + \cosh\Phi x^2 \tag{45}$$

$$= \underline{-\sinh\Phi\frac{1}{\sqrt{2}}(x^+ + x^-) + \cosh\Phi x^2} \tag{46}$$