Problem 9

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

	e	a	b	c
e				
a		not	possible	
b				
c				

	e	a	b	c
e	e	a	b	c
a	a	c	e	b
b	b	e	c	a
c	c	b	a	e

- They are not isomorphic
- The first one is the Vierergroup of Klein, the smallest non cyclic group.

Problem 10

If we apply the Lagrange Theorem to the cyclic group generated by an element $g \in G, g \neq e$ we see that the order of this group is either 1 or n. We take an element that forms a subgroup with the order n. After *Satz 8* this subgroup, which is identical to the whole grouop, is now isomorphic to $\mathbb{Z}_n$

Problem 11

- $e \in \mathcal{L}$ ✓
- $\mathcal{L} \subset GL(4, \mathbb{R}) \Rightarrow \det \Lambda \neq 0, \Lambda \in \mathcal{L}: \quad \det(\Lambda^\intercal \eta \Lambda) = \det \eta \Rightarrow (\det \Lambda)^2 = 1 \Rightarrow \det \Lambda = \pm 1$ ✓
- $\Lambda \in \mathcal{L}: \quad (\Lambda^\intercal \eta \Lambda) = \eta \Rightarrow \eta = (\Lambda^\intercal)^{-1} \eta \Lambda^{-1}$✓
- $\Lambda_1 \Lambda_2 \in \mathcal{L}: (\Lambda_1 \Lambda_2)^\intercal \eta (\Lambda_1 \Lambda_2) = \eta \Rightarrow \Lambda_2^\intercal (\Lambda_1^\intercal \eta \Lambda_1) \Lambda_2 = \Lambda_2^\intercal \eta \Lambda_2 = \eta$✓

So it is a subgroup with $\det \Lambda = \pm 1$. It is obvious that $\mathcal{L}^+$ is a subgroup of $\mathcal{L}$, but it is still to show that it is an invariant subgroup. We take $\det : \mathcal{L} \to \{-1, 1\}$ as a group homomorphismus. The Kernel from det forms an invariant subgroup and obviously $\mathcal{L}^+$ is that kernel.

Problem 12

For SO(n) which consists of n orthogonal vectors, there must be always an even number of vectors v with $v^\intercal v < 0$, so we get:

$$\frac{n}{2} = \frac{n!}{(n-2)!2!} = \frac{n(n-1)}{2} \tag{1}$$

For SU(n) there more options...and somehow we get $(n+1)(n-1)$

$$\frac{n(n-1)}{2} = n^2 - 1 \tag{2}$$

$$0 = n^2 + n - 2 = (n-1)(n+2) \tag{3}$$

So the only valid dimensions is $n = 1$.