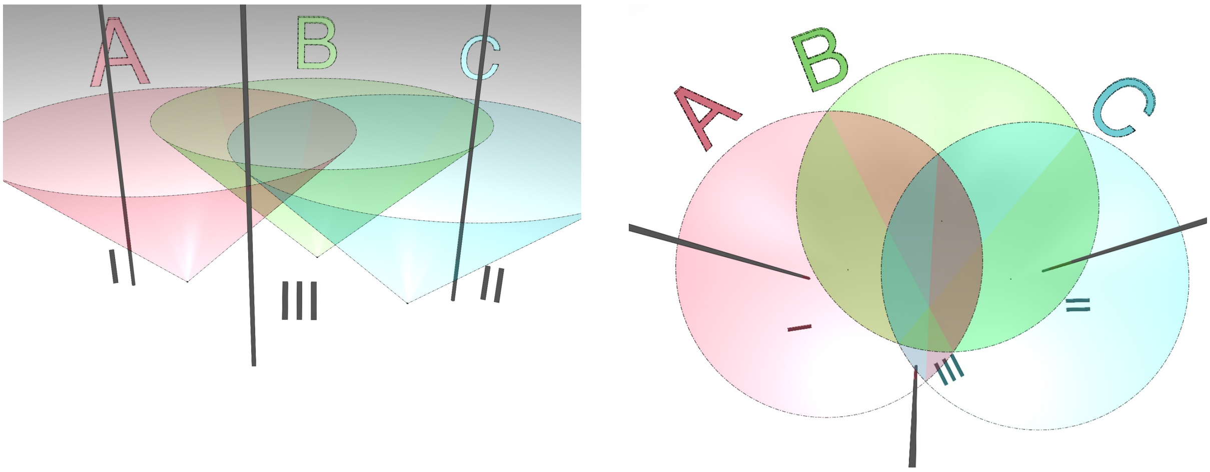
Problem 1

We look at a plain in 3D space, ignoring the z-component and display the time in z-direction. As seen in the picture, the observer I (resting, so displayed as a line like the other observers) sees the events in the order ABC (every event has the same t coordinate), observer II will see it in the opposite direction. Observer III has already seen event A (is inside red cone), is about to see event C (next to blue cone) and after that he will see event B (green cone is the furthest away).



Problem 2

The point $(1, 0, -2)$ is parametrized by $\lambda = 1, \mu = 0, \sigma = -1$. So we get the components as:

(a)
- $(1, 3(\lambda-1)^2, -2) \Rightarrow (1, 0, -2)$
- $(-\sin\mu, \cos\mu, 1) \Rightarrow (0, 1, 1)$
- $(2\sigma, 3\sigma^2 + 2\sigma, 2) \Rightarrow (-2, 1, 2)$

(b)
- $f(\lambda) = 2\lambda^2 + (\lambda-1)^6 + 2\lambda(\lambda-1)^3 \Rightarrow \frac{\mathrm{d}f}{\mathrm{d}\lambda} = 4\lambda + 6(\lambda-1)^5 + 2(\lambda-1)^3 + 6\lambda(\lambda-1)^2$
- $f(\mu) = \cos^2\mu + 1 - (\mu-2)\sin\mu \Rightarrow \frac{\mathrm{d}f}{\mathrm{d}\mu} = 2\sin\mu\cos\mu - \mu\sin\mu + \cos\mu(\mu-2)$
- $f(\sigma) = 2\sigma^4 + (\sigma^3+\sigma^2)^2 - 2(\sigma^4+\sigma^3) \Rightarrow \frac{\mathrm{d}f}{\mathrm{d}\sigma} = 8\sigma^3 + 2(\sigma^3+\sigma^2)(3\sigma^2+2\sigma) - 2(4\sigma^3+3\sigma^2)$

Problem 3

For a boost we write: $t' = (t - vx)\gamma$. With $c = 1$ we have in the inertial frame $\Delta t = \Delta x = \lambda$ as the temporal and spatial distance. Now the boosted observer sees:

$$\lambda' = \Delta t' = \gamma(1-v)\lambda = \sqrt{\frac{(1-v)^2}{(1+v)(1-v)}}\lambda = \sqrt{\frac{1-v}{1+v}}\lambda \tag{1}$$

Problem 4

(a) This is quite easy to proof if one thinks of it geometrically. It is possible to choose a coordinate system in a way, that a vector X^μ can be written as $X^\mu = (a, b, 0, 0)$ so it is possible to only look at the $x - t-$plain. The amount of timelike vectors ($|a| > |b|$) form a cone along the t-axis without

a shell. The amount of lightlike vectors ($|a| = |b|$) form the shell of that cone. All spacelike vectors ($|b| > |a|$) lie in the rest of that whole space.
Obviously each space is not closed, as it is possible to get e.g. from two timelike vectors $(1, 0.1, 0, 0)$, $(-1, 0.1, 0, 0)$ to a spacelike vector $(0, 0.2, 0, 0)$. This is also true for all other vectors.

(b)
- There are spacelike vectors, that are orthogonal to a timelike vector, but they are not a timelike vector itself:

$$t = (1, 1, 0, 0), \quad t_\mu t^\mu = 0, \qquad s = (1, 1, 1, 0), \quad s_0^2 < s_i s^i, \qquad t_\mu s^\mu = 0 \tag{2}$$

- Non spacelike vectors that are orthogonal to a timelike vector, must be a timelike vector: We choose our coordinate system in that way, that the timelike vector can be written as $t = (t_0, \pm t_0, 0, 0)$. So for a vector $o = (o_0, o_1, o_2, o_3)$ to be orthogonal, we get:

$$t_\mu o^\mu = t_0 o_0 \mp t_0 o_1 \stackrel{!}{=} 0 \Rightarrow o_1 = \pm o_0 \tag{3}$$

  So o can only be a timelike vector (with $o_2 = o_3 = 0$), or a spacelike vector ($o_2, o_3 \neq 0$, see example above).

(c) Looking for sets $X_i = (x^0, x^1, x^2, x^3)$ with $X_i \neq \lambda X_j, i \neq j, \lambda \in \mathbb{R}$ and:

$$X_i^\intercal \eta X_i = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = 0 \tag{4}$$

We get:

$$X_1 = (1, 1, 0, 0), \quad X_2 = (1, 0, 1, 0), \quad X_3 = (1, 0, 0, 1), \quad X_4 = (\sqrt{2}, -1, 1, 0) \tag{5}$$