

**Problem 2**

(a)  $E = \gamma m \Rightarrow \gamma = \frac{E}{m}$

The lifetime of the muon in its restframe is given, so we get the observed lifetime as:

$$\Delta t = \gamma \Delta \tau = \frac{E}{m} 2.1910^{-6} \text{ s} \approx 0.0207 \text{ s}$$

(b) The muon travels (in radians)  $\Delta \varphi = \frac{u}{r}$ , with the radius  $r$  and the travelled distance  $u = vt$  of the muon.  $t$  is known, we get  $v$  from:

$$E^2 = m^2 + p^2 \Rightarrow mv = p = \sqrt{E^2 - m^2} \Rightarrow v = \sqrt{\frac{E^2}{m^2} - 1} \quad (1)$$

$$\Rightarrow \Delta \varphi = \frac{\sqrt{\frac{E^2}{m^2} - 1} \cdot \gamma c \Delta \tau}{r} = \frac{\sqrt{\left(\frac{1000}{0.106}\right)^2 - 1} \frac{1000}{0.106} \cdot 3 \cdot 10^8 \frac{\text{m}}{\text{s}} 2.19 \cdot 10^{-6} \text{ s}}{500 \text{ m}} \approx 1.17 \cdot 10^8 \quad (2)$$

**Problem 3**

(a)  $X^{\mu}_{\nu'} = \eta_{\nu'\nu} X^{\mu\nu} = \begin{pmatrix} -2 & 0 & 1 & -1 \\ 1 & 0 & 3 & 2 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & -2 \end{pmatrix}$

(b)  $X_{\mu'}^{\nu} = \eta_{\mu'\mu} X^{\mu\nu} = \begin{pmatrix} -2 & 0 & -1 & 1 \\ -1 & 0 & 3 & 2 \\ -1 & 1 & 0 & 0 \\ -2 & 1 & 1 & -2 \end{pmatrix}$

(c)  $X^{(\mu\nu)} = \frac{1}{2} (X^{\mu\nu} + X^{\nu\mu}) = \begin{pmatrix} 2 & -\frac{1}{2} & 0 & -\frac{3}{2} \\ -\frac{1}{2} & 0 & 2 & \frac{3}{2} \\ 0 & 2 & 0 & \frac{1}{2} \\ -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} & -2 \end{pmatrix}$

(d)  $X_{[\mu\nu]} = \frac{1}{2} (X_{\mu\nu} - X_{\nu\mu}) = \frac{1}{2} \eta_{\mu'\mu} \eta_{\nu'\nu} (X^{\mu\nu} - X^{\nu\mu}) = \eta_{\mu'\mu} \eta_{\nu'\nu} X^{[\mu'\nu']}$   
$$= \eta_{\mu'\mu} \eta_{\nu'\nu} \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & \frac{1}{2} \\ -1 & -1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} & -1 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 1 & \frac{1}{2} \\ 1 & -1 & 0 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

(e)  $X^{\lambda}_{\lambda} = \eta_{\mu\nu} X^{\mu\nu} = -4$

(f)  $V^{\mu} V_{\mu} = \eta_{\mu\nu} V^{\mu} V^{\nu} = -(-1)^2 + 2^2 + 0^2 + 2^2 = 7$

(g)  $V_{\mu} X^{\mu\nu} = \eta_{\mu\mu'} V^{\mu'} X^{\mu\nu} = (4, -2, 5, 7)$