$$\hat{\vec{S}}^2 = \left(\hat{\vec{S}}^{(1)}\right)^2 + \left(\hat{\vec{S}}^{(2)}\right)^2 + 2\hat{\vec{S}}^{(1)}\hat{\vec{S}}^{(2)} \tag{1}$$

$$= \frac{3}{2}\hbar^2\hat{1} + 2\hat{S}_3^{(1)}\hat{S}_3^{(2)} + \hat{S}_+^{(1)}\hat{S}_-^{(2)} + \hat{S}_-^{(1)}\hat{S}_+^{(2)} \tag{2}$$

$$\rightarrow \ |\uparrow\uparrow\rangle = |\underbrace{1}_{j}\ \underbrace{1}_{m}\rangle \quad (j=1, m=+1) \tag{3}$$

$$\rightarrow \ |\downarrow\downarrow\rangle = |1\ -1\rangle \quad (j=1, m=-1) \tag{4}$$

$$\hat{\vec{J}}^2|jm\rangle = \hbar^2 \underbrace{j(j+1)}_{1(1+1)}|jm\rangle \quad \hat{J}_z|jm\rangle = \hbar m|jm\rangle \tag{5}$$

$$\hat{\vec{S}}^2|\uparrow\downarrow\rangle = \left(\frac{3}{2}\hbar^2 - 2\frac{\hbar}{2}\frac{\hbar}{2}\right)|\uparrow\downarrow\rangle + \hbar^2|\downarrow\uparrow\rangle \tag{6}$$

$$= \hbar^2\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \tag{7}$$

$$\hat{\vec{S}}^2|\downarrow\uparrow\rangle = \hbar^2\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) \tag{8}$$

$$\hat{\vec{S}}^2\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = 2\hbar^2\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \quad (j=1, m=0)\text{-Zustand} \tag{9}$$

$\mathbf{j = 1}$

$$|1\ 1\rangle = |\uparrow\uparrow\rangle \tag{10}$$

$$|1\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \tag{11}$$

$$|1\ -1\rangle = |\downarrow\downarrow\rangle \tag{12}$$

Beweis von Gleichung 2:
1. Variante:

$$\left(\hat{\vec{S}}^{(i)}\right)^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle \tag{13}$$

$$= \frac{3}{4}\hbar^2|jm\rangle, \qquad j = \frac{1}{2} \tag{14}$$

$$\rightarrow \left(\hat{\vec{S}}^{(1)}\right)^2 + \left(\hat{\vec{S}}^{(2)}\right)^2 = \frac{3}{2}\hbar^2\hat{1} \tag{15}$$

$$2\hat{\vec{S}}^{(1)}\hat{\vec{S}}^{(2)} = 2\hat{S}_3^{(1)}\hat{S}_3^{(2)} + 2\hat{S}_1^{(1)}\hat{S}_1^{(2)} + 2\hat{S}_2^{(1)}\hat{S}_2^{(2)} \tag{16}$$

$$\hat{S}_+^{(1)}\hat{S}_-^{(2)} = \left(\hat{S}_x^{(1)} + \imath\hat{S}_y^{(1)}\right)\left(\hat{S}_x^{(2)} - \imath\hat{S}_y^{(2)}\right) = \hat{S}_x^{(1)}\hat{S}_x^{(2)} + \hat{S}_y^{(1)}\hat{S}_y^{(2)} + \imath\left(\hat{S}_y^{(1)}\hat{S}_x^{(2)} - \hat{S}_x^{(1)}\hat{S}_y^{(2)}\right) \tag{17}$$

$$\hat{S}_-^{(1)}\hat{S}_+^{(2)} = \hat{S}_x^{(1)}\hat{S}_x^{(2)} + \hat{S}_y^{(1)}\hat{S}_y^{(2)} + \imath\left(\hat{S}_x^{(1)}\hat{S}_y^{(2)} - \hat{S}_y^{(1)}\hat{S}_x^{(2)}\right) \tag{18}$$

2.Variante:

$$\hat{J}_\pm := \hat{J}_x \pm \imath\hat{J}_y \tag{19}$$

$$\hat{J}_\pm|jm\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j, m\pm 1\rangle \tag{20}$$

Anfang: $m_1 = \frac{1}{2}, m_2 = \frac{1}{2} \rightarrow m = 1, j = 1:$

$$|j\ m\rangle = |\uparrow\uparrow\rangle \tag{21}$$

$$\hat{J}_-|1\ 1\rangle = \hbar\sqrt{1(1+1) - 1(1-1)}|1, 1-1\rangle = \hbar\sqrt{2}|1, 0\rangle \tag{22}$$

$$\hat{J}_-|1, 1\rangle = \hat{J}_-|\uparrow\uparrow\rangle = \hat{J}_-^{(1)}|\uparrow\uparrow\rangle + \hat{J}_-^{(2)}|\uparrow\uparrow\rangle = \hbar\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) \tag{23}$$

$$\rightarrow |1\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \tag{24}$$

$$\hat{J}_-|1\ 0\rangle = \sqrt{2}\hbar|1,-1\rangle \tag{25}$$

$$\hat{J}_-|1\ 0\rangle = \frac{1}{\sqrt{2}}\left[\left(\hat{J}_-^{(1)}+\hat{J}_-^{(2)}\right)|\uparrow\downarrow\rangle + \left(\hat{J}_-^{(1)}+\hat{J}_-^{(2)}\right)|\downarrow\uparrow\rangle\right] \tag{26}$$

$$= \frac{1}{\sqrt{2}}\left[\hat{J}_-^{(1)}|\uparrow\downarrow\rangle + \hat{J}_-^{(2)}|\downarrow\uparrow\rangle\right] \tag{27}$$

$$= \frac{1}{\sqrt{2}}\hbar\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)-\frac{1}{2}\left(\frac{1}{2}-1\right)}\left[|\downarrow\downarrow\rangle + |\downarrow\downarrow\rangle\right] \tag{28}$$

$$= \sqrt{2}\hbar|\downarrow\downarrow\rangle \tag{29}$$

$$\rightarrow |1,-1\rangle = |\downarrow\downarrow\rangle \tag{30}$$

Darstellung von $|0,0\rangle$?

- wissen: $|1\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$
- Ansatz: $|0\ 0\rangle = c_1|\uparrow\downarrow\rangle + c_2|\downarrow\uparrow\rangle$

  Orthogonalität:

$$\langle 1\ 0|0\ 0\rangle = 0 \tag{31}$$

$$\rightarrow \frac{1}{\sqrt{2}}(c_1+c_2) = 0 \rightarrow c_2 = -c_1 \tag{32}$$

$$\rightarrow |0\ 0\rangle = c_1\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \tag{33}$$

  Normierung:

$$\langle 0\ 0|0\ 0\rangle = 1 \tag{34}$$

$$|c_1|^2(1+1) = 1 \rightarrow c_1 = \frac{1}{\sqrt{2}} \quad \text{mögliche Wahl} \tag{35}$$

  also:

$$|0\ 0\rangle = \frac{1}{\sqrt{2}}\left(\ \langle\uparrow\downarrow| - |\downarrow\uparrow\rangle\ \right) \tag{36}$$

Besprechung der Übungsaufgabe

Aufgabe3: passt

Aufgabe4a: Spin 0 bedeutet, j=m=0, daher ist die Spinwellenfunktion irrelevant, reduziert sich auf Symmetrieforderung für die Hermite-Polynome $\rightarrow n_2$ gerade.

Aufgabe4b: passt