Problem 1

From the line element one obtains the metric and consequently the Christoffel symbols, which yields by inserting them in the geodetic equation:

$$t'' + H\mathrm{e}^{2Ht}\left(x'^2 + y'^2 + z'^2\right) = 0, \tag{1}$$
$$x'' + 2Hx't' = 0, \tag{2}$$
$$y'' + 2Hy't' = 0, \tag{3}$$
$$z'' + 2Hz't' = 0, \tag{4}$$

where $' = \frac{\mathrm{d}}{\mathrm{d}\lambda}$.

(2)-(4) can be solved with the common ansatz $x' = C\mathrm{e}^{\alpha(\lambda)}$:

$$\alpha' + 2Ht' = 0 \quad \Rightarrow \quad x' = C\mathrm{e}^{-2Ht}. \tag{5}$$

Inserting the last line into (1) yields:

$$t'' + 3C^2H\mathrm{e}^{-2Ht} = 0, \qquad t'' = \frac{\mathrm{d}t'}{\mathrm{d}\lambda} = \frac{\mathrm{d}t'}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\lambda} = \frac{\mathrm{d}t'}{\mathrm{d}t}t' \tag{6}$$
$$t'\,\mathrm{d}t' = -3C^2H\mathrm{e}^{-2Ht}\,\mathrm{d}t, \qquad \Rightarrow \left|\int\right. \tag{7}$$
$$\frac{1}{2}t'^2 = \frac{3C^2H}{2H}\mathrm{e}^{-2Ht} \tag{8}$$
$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = \pm\sqrt{3C^2}\mathrm{e}^{-Ht} \tag{9}$$
$$\mathrm{e}^{Ht}H\frac{\mathrm{d}t}{\mathrm{d}\lambda} = \pm\sqrt{3C^2}H, \quad D = \sqrt{3C^2} \tag{10}$$
$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathrm{e}^{Ht} = \pm DH \tag{11}$$
$$\mathrm{e}^{Ht} = \pm DH\lambda + E, \qquad D, E = \text{const.} \tag{12}$$
$$t = \frac{1}{H}\log\left(\pm DH\lambda + E\right) \tag{13}$$

Obviously $t \to -\infty$ if the argument of log vanishes.