Aufgabe 10

$$\begin{aligned}\hat{\psi}(\xi) &= \sum_i \varphi_i(\xi)\hat{\mathrm{a}}_i, \quad \hat{\psi}^\dagger(\xi) = \sum_i \varphi_i^\dagger(\xi)\hat{\mathrm{a}}_i^\dagger, \quad \hat{\mathrm{N}}_i = \hat{\mathrm{a}}_i^\dagger\hat{\mathrm{a}}_i \\ \hat{\psi}^\dagger(\xi)\hat{\psi}(\xi) &= \sum_i \langle\varphi_i(\xi)|\varphi_i(\xi)\rangle\hat{\mathrm{a}}_i^\dagger\hat{\mathrm{a}}_i \\ &= \sum_i \hat{\mathrm{N}}_i|\varphi_i(\xi)|^2\end{aligned}$$

$|n_0, n_1, n_2, \ldots\rangle$ ist Eigenfunktion zu $\hat{\mathrm{N}}_i$ mit Eigenwert n_i, dabei ist zusätzlich $|n_0, n_1, n_2, \ldots\rangle$ normiert, somit gelangt man zu:

$$\begin{aligned}&\left\langle n_0, n_1, n_2, \ldots \middle| \hat{\psi}^\dagger(\xi)\hat{\psi}(\xi) \middle| n_0, n_1, n_2, \ldots \right\rangle \\ &\left\langle n_0, n_1, n_2, \ldots \middle| \sum_i \hat{\mathrm{N}}_i|\varphi_i(\xi)|^2 \middle| n_0, n_1, n_2, \ldots \right\rangle \\ &= \sum_i n_i|\varphi_i(\xi)|^2\end{aligned}$$

Aufgabe 11

Starte mit:

$$\hat{\mathrm{j}}(\vec{x}) = \frac{1}{2}\sum_i \left[\frac{\hat{\vec{\mathrm{p}}}_i}{m}\delta(\vec{x} - \hat{\mathrm{x}}_i) + \delta(\vec{x} - \hat{\mathrm{x}}_i)\frac{\hat{\vec{\mathrm{p}}}_i}{m}\right]$$

Benutze $\hat{\vec{\mathrm{p}}} = -\imath\hbar\vec{\nabla}$

$$\begin{aligned}&= -\frac{\imath\hbar}{2}\sum_i \left[\frac{\vec{\nabla}_i}{m}\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i) + \delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\frac{\vec{\nabla}_i}{m}\right] \\ &= -\frac{\imath\hbar}{2}\int \mathrm{d}\xi\hat{\psi}^\dagger(\xi)\left[\frac{\vec{\nabla}_i}{m}\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i) + \delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\frac{\vec{\nabla}_i}{m}\right]\hat{\psi}(\xi) \\ &= -\frac{\imath\hbar}{2m}\sum_{m=-s}^{m=s}\left[\int \mathrm{d}\vec{x}\ \hat{\psi}^\dagger(\vec{x})\vec{\nabla}_i\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\hat{\psi}(\vec{x}) + \int \mathrm{d}\vec{x}\ \hat{\psi}^\dagger(\vec{x})\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\vec{\nabla}_i\hat{\psi}(\vec{x})\right] \\ &= -\frac{\imath\hbar}{2m}\sum_{m=-s}^{m=s}\left[\underbrace{\left[\hat{\psi}^\dagger(\vec{x})\vec{\nabla}_i\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\hat{\psi}(\vec{x})\right]_{\infty}^{\infty}}_{=0} - \int \mathrm{d}\vec{x}\ \vec{\nabla}_i\hat{\psi}^\dagger(\vec{x})\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\hat{\psi}(\vec{x}) + \int \mathrm{d}\vec{x}\ \hat{\psi}^\dagger(\vec{x})\delta(\vec{x} - \hat{\vec{\mathrm{x}}}_i)\vec{\nabla}_i\hat{\psi}(\vec{x})\right] \\ &= -\frac{\imath\hbar}{2m}\sum_{m=-s}^{m=s}\left[\hat{\psi}^\dagger(\vec{x})\vec{\nabla}_i\hat{\psi}(\vec{x}) - \vec{\nabla}_i\hat{\psi}^\dagger(\vec{x})\hat{\psi}(\vec{x})\right] \\ &= -\frac{\imath\hbar}{2m}\left[\hat{\psi}^\dagger(\xi)\vec{\nabla}_i\hat{\psi}(\xi) - \vec{\nabla}_i\hat{\psi}^\dagger(\xi)\hat{\psi}(\xi)\right]\end{aligned}$$