Aufgabe 17

a)

Ein Gleichgewicht liegt vor, wenn Gibbsche Potential ein Minimum angenommen hat, dabei sind die chemischen Potentiale der Reaktionspartner gleich. Definiere $\nu = -\sum_i \nu_i$, es ergibt sich nun:

$$\sum_i \nu_i \log n_i + \nu \log n = -\frac{1}{RT} \sum_i \nu_i g_i(T,p)$$

und wie kriegt man jetzt RT aus dem Logarithmus raus? und was hat das mit dem g zu tun?

Aufgabe 18

a)

$$\mathrm{d}U = \delta Q + H\,\mathrm{d}M \to \delta Q = \mathrm{d}U - H\,\mathrm{d}M \tag{1}$$

$$U(H,T) \to \mathrm{d}U = \left(\frac{\partial U}{\partial H}\right)_T \mathrm{d}H + \left(\frac{\partial U}{\partial T}\right)_H \mathrm{d}T \to \frac{\mathrm{d}H}{\mathrm{d}T} = 0 \tag{2}$$

$$U(M(T,H),T) \to \mathrm{d}U = \left(\frac{\partial U}{\partial M}\right)_T \mathrm{d}M + \left(\frac{\partial U}{\partial T}\right)_M \mathrm{d}T \to \frac{\mathrm{d}M}{\mathrm{d}T} = \left(\frac{\partial M}{\partial T}\right)_H \tag{3}$$

$$\left(\frac{\partial U}{\partial T}\right)_H = \left(\frac{\partial U}{\partial M}\right)_T \left(\frac{\partial M}{\partial T}\right)_H + \left(\frac{\partial U}{\partial T}\right)_M \tag{4}$$

$$\left(\frac{\delta Q}{\delta T}\right)_H = \left(\frac{\partial U}{\partial T}\right)_H - \left(\frac{\partial (H\,\mathrm{d}M)}{\partial T}\right)_H, \quad \left(\frac{\delta Q}{\delta T}\right)_M = \left(\frac{\partial U}{\partial T}\right)_M - \left(\frac{\partial (H\,\mathrm{d}M)}{\partial T}\right)_M \tag{5}$$

$$\left(\frac{\delta Q}{\delta T}\right)_H - \left(\frac{\delta Q}{\delta T}\right)_M = \left(\frac{\partial U}{\partial T}\right)_H - \left(\frac{\partial U}{\partial T}\right)_M - \left(\frac{\partial (H\,\mathrm{d}M)}{\partial T}\right)_H + \left(\frac{\partial (H\,\mathrm{d}M)}{\partial T}\right)_M \tag{6}$$

$$c_H - c_M = \left(\frac{\delta Q}{\delta T}\right)_H - \left(\frac{\delta Q}{\delta T}\right)_M = \left(\frac{\partial U}{\partial M}\right)_T \left(\frac{\partial M}{\partial T}\right)_H - H\left(\frac{\partial M}{\partial T}\right)_H \tag{7}$$

b)

Magnetisierung M ist eine Funktion von $T \to \left(\frac{\partial U}{\partial M}\right)_T = 0$:

$$c_H - c_M = -H\left(\frac{\partial M}{\partial T}\right)_H = -H\left(-\frac{CH}{T^2}\right) = \frac{1}{C}\frac{H^2C^2}{T^2} = \frac{M^2}{C}$$

Aufgabe 19

a)

$$\frac{\alpha}{2}x^2 - \beta x = \frac{\alpha}{2}x^2 - \beta x + \gamma^2 - \gamma^2 = \left(\sqrt{\frac{\alpha}{2}}x - \gamma\right)^2 - \gamma^2, \quad \beta = 2\sqrt{\frac{\alpha}{2}}\gamma \to \gamma = \frac{\beta}{\sqrt{2\alpha}}$$

$$\int_{-\infty}^{\infty} \exp\left[-\left(\frac{\alpha}{2}x^2 - \beta x\right)\right] \mathrm{d}x = \int_{-\infty}^{\infty} \exp\left[-\left(\sqrt{\frac{\alpha}{2}}x - \gamma\right)^2\right] \exp\left[\frac{\beta^2}{2\alpha}\right] \mathrm{d}x$$
$$= \int_{-\infty}^{\infty} \exp\left[-\left(\sqrt{\frac{\alpha}{2}}x - \gamma\right)^2\right] \exp\left[\frac{\beta^2}{2\alpha}\right] \mathrm{d}x, \quad \sqrt{\frac{\alpha}{2}}x \to x,\, \mathrm{d}x \to \sqrt{\frac{2}{\alpha}}\,\mathrm{d}x$$
$$= \int_{-\infty}^{\infty} \exp\left[-(x-\gamma)^2\right] \mathrm{d}x \cdot \sqrt{\frac{2}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right], \quad (x-\gamma) \to x,\, \mathrm{d}x \to \mathrm{d}x$$
$$= \int_{-\infty}^{\infty} \exp\left[-x^2\right] \mathrm{d}x \cdot \sqrt{\frac{2}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right]$$
$$= \sqrt{\int_{-\infty}^{\infty} \exp\left[-y^2\right] \mathrm{d}y \cdot \int_{-\infty}^{\infty} \exp\left[-x^2\right] \mathrm{d}x} \cdot \sqrt{\frac{2}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right], \quad x \to r\cos\phi, y \to r\sin\phi$$
$$= \sqrt{\int_0^{\infty}\int_0^{2\pi} \exp\left[-r^2\right] r\,\mathrm{d}r\,\mathrm{d}\phi} \cdot \sqrt{\frac{2}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right], \quad \frac{\mathrm{d}}{\mathrm{d}r}\exp\left[-r^2\right] = -2r\exp\left[-r^2\right]$$
$$= \sqrt{2\pi \int_0^{\infty} -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r} \exp\left[-r^2\right] \mathrm{d}r} \cdot \sqrt{\frac{2}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right],$$
$$= \sqrt{2\pi \int_0^{\infty} -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r} \exp\left[-r^2\right] \mathrm{d}r}$$
$$= \sqrt{\frac{2\pi}{\alpha}} \exp\left[\frac{\beta^2}{2\alpha}\right]$$

b)

$$\alpha \to \frac{\alpha}{2}, \quad x^n \exp\left[-\frac{\alpha}{2}x^2\right] = \frac{\mathrm{d}^n}{\mathrm{d}\beta^n} \exp\left[-\left(\frac{\alpha}{2}x^2 - \beta x\right)\right]\Big|_{\beta=0}$$
$$\int_{-\infty}^{\infty} x^n \exp\left[-\frac{\alpha}{2}x^2\right] \mathrm{d}x = \int_{-\infty}^{\infty} \frac{\mathrm{d}^n}{\mathrm{d}\beta^n} \exp\left[-\left(\frac{\alpha}{2}x^2 - \beta x\right)\right]\Big|_{\beta=0} \mathrm{d}x$$
$$= \sqrt{\frac{2\pi}{\alpha}} \frac{\mathrm{d}^n}{\mathrm{d}\beta^n} \exp\left[\frac{\beta^2}{2\alpha}\right]\Big|_{\beta=0} \overset{\alpha\to 2\alpha}{=} \begin{cases} 0, & n \text{ ungerade} \\ \sqrt{\frac{\pi}{\alpha}}\left(\frac{1}{2\alpha}\right)^m \frac{(2n'+1)!}{2^{n'}n'!}, & n \text{ gerade}, n' = n-2, m = \frac{1}{2}n \end{cases}$$

Aufgabe 20

Allgemein gilt:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\partial\rho}{\partial t} + \sum_i^{3N} \frac{\partial\rho}{\partial q_i}\frac{\partial q_i}{\partial t} + \sum_i^{3N} \frac{\partial\rho}{\partial p_i}\frac{\partial p_i}{\partial t} = \frac{\partial\rho}{\partial t} + X\nabla\rho$$

Da $X = (\dot{q}_1, ..., \dot{q}_{3N}, \dot{p}_1, ...\dot{p}_{3N}) = (\frac{\partial H}{\partial p_1}, ..., \frac{\partial H}{\partial p_{3N}}, -\frac{\partial H}{\partial q_1}, ..., -\frac{\partial H}{\partial q_{3N}})$: bildet man davon die Divergenz $\nabla = (\frac{\partial}{\partial q_1}, ..., \frac{\partial}{\partial q_{3N}}, -\frac{\partial}{\partial p_1}, ..., -\frac{\partial}{\partial p_{3N}})$, verschwindet diese auf Grund der gemischten partiellen Ableitungen.

Somit folgt:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\partial \rho}{\partial t} + \nabla(X\rho)$$

Die totale Zeitableitung verschwindet nun aufgrund der Kontinuitätsgleichung: $\frac{\partial \rho}{\partial t} - \nabla(\vec{j}) = \frac{\partial \rho}{\partial t} - \nabla(\rho X) = 0$, denn $\vec{j} = \sum_i^{3N} \frac{\partial \rho}{\partial q_i}\frac{\partial q_i}{\partial t} + \sum_i^{3N} \frac{\partial \rho}{\partial p_i}\frac{\partial p_i}{\partial t}$.