Physics of Planetary Systems — Exercises

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Solutions to Set 9

Problem 9.1

A depth of

$$0.00064 = (R_{\rm p}/R_*)^2$$

corresponds to a radius

$$R_{\rm p} = 1.77 \times 10^4 \text{ km} = 2.78\ R_\oplus.$$

Given an Earth density, the mass of this planet would be

$$M_{\rm p} = 1.29 \times 10^{29} \text{ g} = 21.5\ M_\oplus = 0.07\ M_{\rm Jup}.$$

From

$$A = \sqrt[3]{\frac{2\pi G}{P(M_* + M_{\rm p})^2}} M_{\rm p} \sin i$$

with the period $P = 0.65$ days, a stellar mass $M_* = M_\odot$, and $i = 90^\circ$, we get a velocity amplitude

$$A = 15.8 \text{ m/s}.$$

Problem 9.2

The following steps should be performed:

- Take on/off photometry, i.e. observe the star in transit and out-of-transit to make sure that the target star is the one causing the transit.
- Take a spectrum to determine that the star is a dwarf and not a giant star.
- Take high resolution imaging to exclude nearby, faint background stars
- Take infrared spectroscopy to make sure that a late-type binary star (triple system) is not causing the transit
- Take a few radial velocity measurements to exclude a grazing eclipse by a binary.

If all of these tests are passed, the only possible explanation is that the star is transited by an object with a radius of 2.78 $R_\oplus$. There are only 3 astronomical objects with such a small radius: a white dwarf, a neutron star, and a planet. The first two would cause a radial velocity variation of tens of km/s, easily excluded with a few RV measurements at low precision.

Problem 9.3

If the material from both sides of the gap can reach the gap center within one orbital period, the gap is closed. Thus, we have to equate the time the planet needs to reach the same position relative to the gas again with the time needed for the gas to traverse a distance equal to the planet's Hill radius r_H:

$$\frac{2\pi r}{v_\mathrm{rel}} = \frac{r_\mathrm{H}}{v_\mathrm{fill}},$$

where v_fill is the speed at which the gas can refill the gap and v_rel is the difference between the tangential velocities of gas and embryo:

$$v_\mathrm{rel} = v_\mathrm{K} - v_\mathrm{gas} = v_\mathrm{K} \cdot (1 - \sqrt{1-2\eta}) \approx \eta v_\mathrm{K}, \tag{1}$$

with $\eta \equiv c_\mathrm{s}^2/v_\mathrm{K}^2$. Using

$$r_\mathrm{H} = r\left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_*}\right)^{1/3},$$

we arive at

$$\frac{2\pi r}{v_\mathrm{rel}} = \frac{r}{v_\mathrm{fill}}\left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_*}\right)^{1/3},$$

and solving for $\mathscr{M}_\mathrm{p}$ leads to

$$\mathscr{M}_\mathrm{p} = 3\mathscr{M}_*\left(\frac{2\pi v_\mathrm{fill}}{v_\mathrm{rel}}\right)^3.$$

Now, we can assume a filling velocity that equals the radial drift velocity:

$$v_\mathrm{fill} = \frac{3\nu}{2r} = \frac{3\alpha c_\mathrm{s}^2}{2r\Omega_\mathrm{K}} = \frac{3\alpha c_\mathrm{s}^2}{2v_\mathrm{K}}, \tag{2}$$

from which we obtain

$$\mathscr{M}_\mathrm{p} = 3\mathscr{M}_*\left(\frac{3\pi\alpha c_\mathrm{s}^2}{\eta v_\mathrm{K}^2}\right)^3 = 3\mathscr{M}_*(3\pi\alpha)^3 \approx 2500\alpha^3\mathscr{M}_*.$$

Assuming $\alpha = 0.001\ldots0.01$ and $\mathscr{M}_* = M_\odot$ leads to

$$\begin{aligned}\mathscr{M}_\mathrm{p} &= 2.5\times10^{-6}\mathscr{M}_*\ldots2.5\times10^{-3}\mathscr{M}_* \\ &\approx 1\ M_\oplus\ldots2\ M_\mathrm{Jup}.\end{aligned}$$

Problem 9.4

In case of a massive planet, the gap stays more or less cleared and the amount of material that streams into the gap per orbital period P is given by

$$\Delta\mathscr{M} = 4\pi r h \rho_\mathrm{gas} v_\mathrm{fill} P,$$

where h is the scale height of the disk and $4\pi rh$ the "surface" of the gap. Using $\rho_\mathrm{gas} = \Sigma_\mathrm{gas}/h$ (where $h = c_\mathrm{s}/\Omega_\mathrm{K}$), we find

$$\Delta\mathscr{M}_\mathrm{p} = 4\pi r\Sigma_\mathrm{gas} v_\mathrm{fill} P.$$

Assuming now $\Sigma_\mathrm{gas} = 100\Sigma_\mathrm{MMSN} \approx 10000$ kg m^{-2}, $v_\mathrm{fill} \approx 2$ m s^{-1}, $P = 3.16\times10^7$ s, and $r = 1.5\times10^{11}$ m, we obtain

$$\Delta\mathscr{M}_\mathrm{p} = 1.2\times10^{24}\ \mathrm{kg} = 0.2\ \mathscr{M}_\oplus,$$

which is quite a lot: that's 1 M_Jup in 1500 years.

A low-mass planet, on the other hand, always finds a filled gap ahead of it and can eat all the material within this gap that that comes closer than one hill radius, i.e. everything that streams through the cross section defined by its Hill radius:

$$\begin{aligned}
\Delta\mathscr{M}_\mathrm{p} &= \pi r_\mathrm{H}^2 \rho_\mathrm{gas} v_\mathrm{rel} P \\
&= \pi r^2 \left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_\odot}\right)^{2/3} \frac{\Sigma_\mathrm{gas}}{h} v_\mathrm{rel} P \\
&= \pi r^2 \left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_\odot}\right)^{2/3} \frac{\overbrace{\Omega_\mathrm{K}}^{=v_\mathrm{K}/r} P \Sigma_\mathrm{gas} \overbrace{v_\mathrm{rel}}^{=\eta v_\mathrm{K}}}{c_\mathrm{s}} \\
&= \pi r \left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_\odot}\right)^{2/3} \frac{P \Sigma_\mathrm{gas} \eta v_\mathrm{K}^2}{c_\mathrm{s}} \\
&= \pi r \left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_\odot}\right)^{2/3} P \Sigma_\mathrm{gas} c_\mathrm{s}.
\end{aligned}$$

With $P = 3.16 \times 10^7$ s, $\Sigma = 10000$ kg/cm^2, and $c_\mathrm{s} = 1000$ m/s, we end up with

$$\Delta\mathscr{M}_\mathrm{p} \approx 1.5 \times 10^{26}\ \mathrm{kg} \left(\frac{\mathscr{M}_\mathrm{p}}{3\mathscr{M}_\odot}\right)^{2/3} \approx 12\ \mathscr{M}_\oplus \left(\frac{\mathscr{M}_\mathrm{p}}{\mathscr{M}_\odot}\right)^{2/3} \approx 2.5 \times 10^{-3}\ \mathscr{M}_\oplus \left(\frac{\mathscr{M}_\mathrm{p}}{\mathscr{M}_\oplus}\right)^{2/3}$$