# Physics of Planetary Systems — Exercises

Astrophysikalisches Institut und Universitätssternwarte Jena
Thüringer Landessternwarte Tautenburg

2009-06-25

## Solutions to Set 8

### Problem 8.1

First, the slope can be estimated by eye, or by plotting and using a ruler. Let us assume a change of $-90$ m/s over a time difference of $39.88\text{ d} - 39.78\text{ d} = 0.1\text{ d} = 2.4\text{ h}$. This is equivalent to a derivative of $-0.0104\text{ m s}^{-2}$.
For circular orbits one can assume a sine function:

$$V = -A\sin\left[\omega(t-t_0)\right] = -A\sin\frac{2\pi(t-t_0)}{P},$$

where $A$ is the amplitude and $P$ is the period. The minus sign stems from the fact that phase 0, by definition, represents the transit time, whereafter the velocity gets negative. Taking the derivative, we obtain

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{2\pi A}{P}\cos\frac{2\pi(t-t_0)}{P}.$$

At transit time, i.e. for $t = t_0$, we have $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{2\pi A}{P} = -0.0104\text{ m s}^{-2}$. Using a period of 2.8 days (convert to seconds!) one should get

$$A = 385\text{ m s}^{-1}.$$

From the amplitude we can get the planet mass from the equation given in the first RV lecture:

$$V_{\mathrm{obs}} = \sqrt[3]{\frac{2\pi G}{P(M_* + M_{\mathrm{p}})^2}}M_{\mathrm{p}}\sin i \qquad \text{or} \qquad V_{\mathrm{obs}} = \frac{28.4 M_{\mathrm{p}}\sin i}{P^{1/3}(M_* + M_{\mathrm{p}})^{2/3}},$$

where $V_{\mathrm{obs}}$ is the amplitude in m/s, $P$ the period in years, $M_{\mathrm{p}}$ the planet mass in Jupiters, and $M_*$ the stellar mass in solar units. Assuming $M_* \gg M_{\mathrm{p}}$ and $\sin i = 1$ (transit!) and solving for the mass of the planet, one gets

$$M_{\mathrm{p}} = \frac{V_{\mathrm{obs}}P^{1/3}(M_* + M_{\mathrm{p}})^{2/3}}{28.4}, \qquad \text{leading to} \qquad M_{\mathrm{p}} = 2.67\ M_{\mathrm{Jup}}.$$

So this is most likely a planet and in fact it now has high priority for follow-up measurements. More RV measurements are planned for next winter so as to get the full RV curve and a better measurement of the mass. The exact answer depends on how you measure the slope, so anything around 2 $M_{\mathrm{Jup}}$ is fine.

### Problem 8.2

The stellar radius is given by

$$R_* = 0.55\tau M_*^{1/3}P^{-1/3},$$

where $M_*$ is in solar mass units, $P$ is the period in days, $\tau = 0.3\ldots0.4$ the transit duration in hours, and $R$ the radius in solar units. Plugging in the values, we obtain

$$R = 1.7\ldots2.3\ R_\odot.$$

This is definitely not a solar-type star. Most likely it is not a good planet candidate. But we have more information: the transit depth. Since $(R_{\mathrm{p}}/R_*)^2 = 0.011$, the planet has a radius

$$R_{\mathrm{p}} = 0.18\ldots0.24\ R_\odot = 1.8\ldots2.4\ R_{\mathrm{Jup}},$$

which is too big for a planet. The chances are the companion is an M dwarf star.

## Problem 8.3

From energy conservation, we find

$$\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{N}}{r_{\mathrm{N},0}}+\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{U}}{r_{\mathrm{U},0}}+\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{J}}{r_{\mathrm{J},0}}=\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{N}}{r_{\mathrm{N},1}}+\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{U}}{r_{\mathrm{U},1}}+\frac{G\mathscr{M}_*\mathscr{M}_\mathrm{J}}{r_{\mathrm{J},1}}$$

and, therefore,

$$r_{\mathrm{J},1}=\mathscr{M}_\mathrm{J}\left[\frac{\mathscr{M}_\mathrm{N}}{r_{\mathrm{N},0}}-\frac{\mathscr{M}_\mathrm{N}}{r_{\mathrm{N},1}}+\frac{\mathscr{M}_\mathrm{U}}{r_{\mathrm{U},0}}-\frac{\mathscr{M}_\mathrm{U}}{r_{\mathrm{U},1}}+\frac{\mathscr{M}_\mathrm{J}}{r_{\mathrm{J},0}}\right]^{-1}.$$

With $r_{\mathrm{J},0}=5$ AU, $r_{\mathrm{U},0}=r_{\mathrm{N},0}=7$ AU, $r_{\mathrm{U},1}=19$ AU, $r_{\mathrm{N},1}=30$ AU, $\mathscr{M}_\mathrm{N}=17\ \mathscr{M}_\oplus$, $\mathscr{M}_\mathrm{U}=14\ \mathscr{M}_\oplus$, and $\mathscr{M}_\mathrm{J}=318\ \mathscr{M}_\oplus$ kg, the result is

$$r_{\mathrm{J},1}=4.76\ \mathrm{AU}.$$

As expected, the change is only slight.

## Problem 8.4

We assume a surface mass density $\Sigma(r)=\Sigma_0(r/r_0)^{-3/2}$ with $\Sigma_0=3$ g cm$^2$ at $r_0=5$ AU $=7.5\times10^{13}$ cm. The mass in the range from 30 to 50 AU is thus given by

$$\begin{aligned} M &= \int_{30\ \mathrm{AU}}^{50\ \mathrm{AU}} 2\pi r\Sigma\,\mathrm{d}r = 4\pi\Sigma_0 r_0^2\left(\frac{r}{r_0}\right)^{1/2}\Bigg|_{r=30\ \mathrm{AU}}^{r=50\ \mathrm{AU}} \\ &= 1.5\times10^{29}\ \mathrm{g}\approx 25\ \mathscr{M}_\oplus. \end{aligned}$$

This result of 25 Earth masses is more than two orders of magnitude higher than the estimated current mass in that region. One reason for this discrepancy might by a strong depletion of material due to a stellar encounter or due to the perturbing gravitational influence of Neptune. And, to begin with, the assumed initial condition as an idealized MMSN might not be valid at all at such distances: every protoplanetary disk “ends” somewhere.