Physics of Planetary Systems — Exercises

Astrophysikalisches Institut und Universitätssternwarte Jena
Thüringer Landessternwarte Tautenburg

2009-06-18

Solutions to Set 7

Problem 7.1

The radial velocity amplitude of an earth in the habitable zone of a G star is about 9 cm/s. The intrinsic variability of a star can have amplitudes of 1 m/s or more. Sources of noise are:

- Stellar oscillations, periods of $\sim$5–10 min, amplitudes of 0.3–0.5 m/s
- Spots due to activity, periods of 4–50 days (i.e. tens of days) and amplitudes of 1–100 m/s, depending on the age of the star and level of activity
- changes in the convection pattern of the star, periods $\sim$10 years, amplitudes $\sim$10 m/s

Problem 7.2

In class, the equation for the equilibrium was given:

$$T_{\rm planet} = T_* \left(\frac{R_*}{d}\right)^{1/2} \left(\frac{1-A}{4}\right)^{1/4}. \tag{1}$$

From Kepler's law, we can compute the planet's orbital semi-major axis: $a = 1.27$ AU. At perihelion, the planet has a distance $d = a(1-e) = 1.27$ AU $(1-0.9) = 0.13$ AU. Assuming an albedo $A = 0.35$, this results in a surface temperature $T_{\rm planet} = 694$ K $= 421°$C. At aphelion, the distance is $a(1+e) = 2.41$ AU and the according temperature is $T = 160$ K $= -113°$C. With temperature extremes of more than 500°C in a time span of 1.4 years, the seasons would indeed be extreme. Thus, not only would life need to live with extremes, as is shown to be possible on earth, but also with extreme changes. Therefore, the planet is probably not habitable, even though its semi-major axis lies in the habitable zone. The eccentricity is simply too high.

Problem 7.3

The 3D equation of mass growth rate is:

$$\frac{\mathrm{d}\mathscr{M}}{\mathrm{d}t} \approx \rho\sigma v_{\rm rel}$$

In 2D, the following changes are needed. First, ρ is replaced by surface density, Σ. Second, the cross section for collision σ without gravitational enhancement is $2s$ instead of πs^2. With gravitational enhancement, it is therefore

$$\sigma = 2s\left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right)^{1/2}$$

instead of

$$\sigma = \pi s^2\left(1 + \frac{v_{\rm esc}^2}{v_{\rm rel}^2}\right).$$

Collecting all results together and using $v_{\text{rel}} \ll v_{\text{esc}}$ as in the 3D case, we obtain the 2D equation of mass growth rate:

$$\frac{\mathrm{d}\mathscr{M}}{\mathrm{d}t} \approx \Sigma \cdot 2s \cdot v_{\text{esc}}$$

or

$$\underline{\underline{\frac{\mathrm{d}\mathscr{M}}{\mathrm{d}t} \propto \Sigma \mathscr{M}^{2/3}}}$$

where the power of the mass in the right-hand side is less than unity, so it is not a runaway growth. The exponent (2/3) is the same as for the oligarchic growth.

Problem 7.4

Obviously, $\sigma = \pi B^2$, where B is the impact parameter of a hyperbola such that the distance of closest approach $r_{\min}$ equals the planetesimal radius s (see Figure).
The (specific) angular momentum of the small planetesimal "at infinity" is $L = v_{\text{rel}} B$, at the closest approach it is $L = v_s s$.
The (specific) energy at these two points is $E = \frac{1}{2} v_{\text{rel}}^2$ and $E = \frac{1}{2} v_s^2 - \frac{G\mathscr{M}}{s}$.
Conservation of L leads to

$$v_{\text{rel}} B = v_s s \tag{2}$$

and conservation of E yields

$$\frac{1}{2} v_{\text{rel}}^2 = \frac{1}{2} v_s^2 - \frac{G\mathscr{M}}{s}. \tag{3}$$

Noting that

$$v_{\text{esc}}^2 = \frac{2G\mathscr{M}}{s},$$

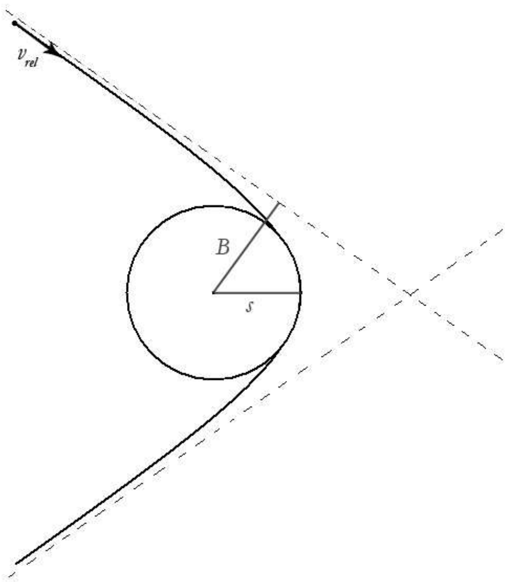
equation (3) can be rewritten as

$$v_s^2 = v_{\text{esc}}^2 + v_{\text{rel}}^2.$$

Equation (2) gives:

$$B^2 = s^2 \left(\frac{v_s}{v_{\text{rel}}} \right)^2 = s^2 \left(1 + \frac{v_{\text{esc}}^2}{v_{\text{rel}}^2} \right).$$

Using $\sigma = \pi B^2$ leads to the desired result.



Problem 7.5

The mass of finished oligarchs is given by

$$\mathscr{M}_{\rm iso} = \frac{(2\pi b\Sigma)^{3/2} r^3}{(3\, M_*)^{1/2}}$$

Assume $b = 10$ and $\Sigma = 10$ g cm^{-2} at 1 AU. Then

$$\begin{aligned}\mathscr{M}_{\rm iso} &= \frac{(2\cdot 3\cdot 10\cdot 10)^{3/2}(1.5\cdot 10^{13})^3}{(3\cdot 2\cdot 10^{33})^{1/2}} \approx \frac{600^{3/2}\cdot 3\cdot 10^{39}}{(60\cdot 10^{32})^{1/2}} \approx \frac{600\cdot 25\cdot 3\cdot 10^{39}}{8\cdot 10^{16}} \approx \frac{600\cdot 10\cdot 10^{39}}{10^{16}} \\ &\approx 6\cdot 10^{26}\ \text{g} \approx 0.1\, \mathscr{M}_\oplus\end{aligned}$$

To get the result at 5 AU, we have to multiply this by $(3/10)^{3/2}$ (to account for difference in Σ) and by $(5/1)^3$ (to account for difference in r). This gives:

$$\mathscr{M}_{\rm iso} \approx 0.1\, \mathscr{M}_\oplus \cdot (3/10)^{3/2} \cdot 5^3 \approx 0.1\, \mathscr{M}_\oplus \cdot 20 \approx 2\, \mathscr{M}_\oplus.$$

The orbital separation of isolated oligarchs is given by

$$\Delta r = b r_{\rm H} = br \left(\frac{\mathscr{M}_{\rm iso}}{3M_*}\right)^{1/3}$$

Numerically, at 1AU

$$\Delta r \approx 10\cdot 1.5\cdot 10^{13} \left(\frac{6\cdot 10^{26}}{3\cdot 2\cdot 10^{33}}\right)^{1/3} \approx 1.5\cdot 10^{14} \left(\frac{1}{10\cdot 10^6}\right)^{1/3} \approx 10^{12}\ \text{cm} \approx 0.07\ \text{AU}$$

To get the result at 5 AU, we have to multiply this by 5 (to account for difference in r) and with $20^{1/3}$ (to account for difference in $\mathscr{M}_{\rm iso}$). This gives:

$$\Delta r \approx 0.07\ \text{AU} \cdot 5 \cdot 2.7 \approx 1\ \text{AU}$$