Physics of Planetary Systems — Exercises

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2009-06-04

Solutions to Set 5

Problem 5.1

The astrometric amplitude is given by

$$\theta = \frac{M_{\text{planet}}}{M_*} \frac{a}{D},$$

where M_{planet} is the mass of the planet, M_* the stellar mass, a the orbital semi-major axis, and D the stellar distance. Again the students have to calculate a based on the M_* and the period P. The result is $a = 1.18$ AU. So the astrometric amplitude is $\theta = \frac{7.4\times0.001\ M_\odot}{1.23\ M_\odot} \cdot \frac{1.18\cdot1.5\times10^{13}\ \text{cm}}{38.5\cdot3.086\times10^{18}\ \text{cm}} = 0.90\times10^{-9}$ radians. We need to convert this to a more interesting unit, i.e. milli-arcseconds. With 1 rad $= 180^\circ/\pi = 648000''/\pi$, we end up with an astrometric perturbation

$$\theta = 0.00018'' = 0.18 \text{ mas}.$$

In other words, astrometry should be able to confirm this planet.

Problem 5.3

The incoming energy per unit time is the cross-section area of a spherical grain times the flux of stellar radiation at a distance r:

$$\pi s^2 \cdot \frac{L_*}{4\pi r^2}$$

or, using the Stefa-Boltzmann equation:

$$\pi s^2 \cdot \frac{\sigma T_*^4 R_*^2}{r^2}$$

The outgoing energy per unit time is related to the particle's total surface area:

$$4\pi s^2 \cdot \sigma T_{\text{dust}}^4$$

In equilibrium both are equal:

$$\pi s^2 \cdot \frac{\sigma T_*^4 R_*^2}{r^2} = 4\pi s^2 \cdot \sigma T_{\text{dust}}^4$$

so that

$$r = \frac{R_*}{2} \left(\frac{T_*}{T_{\text{dust}}} \right)^2$$

or

$$T_{\text{dust}} = T_* \sqrt{\frac{R_*}{2r}}.$$

At the distance of the Earth, we find

$$T_{\text{dust}} = T_\odot \sqrt{\frac{R_\odot}{2r}} \approx 290 \text{ K}.$$

Using $R_* = R_\odot$ and $T_{\text{dust}} = 1500$ K, the sublimation distance is then given by

$$r \approx 8\ R_\odot.$$

Problem 5.2

In class, the expression

$$T_{\text{planet}} = T_* \left(\frac{R_*}{d}\right)^{1/2} \left(\frac{1-A}{4}\right)^{1/4} \tag{1}$$

was derived equating incoming flux in and outgoing flux. A is the albedo, d is the distance of the planet and R_* the radius of the star. Note: this is for a redistribution of heat, i.e. the absorbed flux of the planet gets re-radiated in 4π steradians. For the albedo, we need to find a reasonable value. Some examples are: Jupiter with $A = 0.52$ and Earth with $A = 0.37$. The expectation for hot Jupiters is $A = 0.1$.

The orbital distance, d, can be calculated using Newton's form of Keplers law which depends on the stellar mass:

$$\left(\frac{P}{1\text{ yr}}\right)^2 = \frac{M_\odot}{M_*}\left(\frac{a}{1\text{ AU}}\right)^2,$$

resulting in $d = a = 0.032$ AU $= 4.8\times10^{11}$ cm. Assuming $A = 0.1$, we get an equilibrium temperature

$$T_{\text{planet}} = 1920\text{ K}.$$

If you assume that the planet is tidally locked and that it re-radiates in only 2π steradians, 4 becomes 2 in the equation 1 and

$$T'_{\text{planet}} = 2282\text{ K}.$$

Problem 5.4

From

$$\Gamma = \frac{\rho}{\rho_{\text{dust}}}\frac{c_{\text{s}}}{s}$$

and the boundary condition

$$\Gamma = 2\Omega_{\text{K}},$$

we find

$$s = \frac{\rho}{\rho_{\text{dust}}}\frac{c_{\text{s}}}{2\Omega_{\text{K}}}$$

The sound speed was estimated in one of the previous problems: at 1 AU, $c_s \sim 10^3$ m s^{-1}. For the Kepler frequency we can assume $\Omega_{\text{K}} = \frac{2\pi}{1\text{ yr}} = \frac{2\pi}{3\times10^7\text{ s}}$, and for the dust density $\rho_{\text{dust}} = 1$ g cm^{-3} $= 1000$ kg m^{-3}. Then,

$$s \sim 2.5\text{ m}.$$

Therefore, the boundary is at meter-sized bodies.

Problem 5.5

The acceleration due to gas drag force is proportional to σ/m (the ratio of cross section and mass) and therefore to $1/s$. Thus, the smaller the bodies, the stronger the headwind they experience. But this is true only down to a certain size. Yet smaller particles become more and more strongly coupled with gas, their relative velocity with respect to gas tends to zero, and so does the radial velocity.