Physics of Planetary Systems — Exercises

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2009-05-28

Solutions to Set 4

Problem 4.1

This figure appeared in Dreizler et al. 2003, A&A, 402, 791. This paper should have been rejected by the referee. One should of course be critical that within the errors one can also fit a straight line through all data points. The RV measurements are consistent with no variations. However, the proof against the planet is in the phase. Photometric phase 0 is at transit center. At this phase the star is behind the planet and moving transversely to the observers left (0 radial velocity). After this phase, the star should start moving towards the observer. This is a blue shift in wavelength and by definition should be a negative radial velocity after phase 0. But the radial velocity curve for this star is 180 degrees out of phase from that expected for a transiting planet. This is impossible, so the RV data do not support the planet hypothesis and the paper and press release should have never been published.

Problem 4.2

Assuming a circular orbit, you can derive everything trivially from Kepler's law, or you can use the expression given in class,

$$R_* = 0.55 R_\odot \frac{\tau}{1\ \mathrm{h}} \left(\frac{M_*}{M_\odot} \frac{1\ \mathrm{day}}{P} \right)^{1/3}$$

where M_* is the stellar mass, P the planet's period, τ the transit duration, and R_* the resulting stellar radius. (We neglect the radius of the planet for now.) Reading off the graph, the transit duration is approximately $0.04\ \mathrm{d} = 3456\ \mathrm{s} = 0.96\ \mathrm{h}$. Assuming $M_* = M_\odot$, one gets $R_* = 0.38\ R_\odot$, a radius consistent with an M dwarf.

One can refine the radius by assuming that the stellar mass is now $M_* = 0.4\ M_\odot$, more appropriate for an M dwarf. In that case, the resulting stellar radius is $R_* = 0.28\ R_\odot$

Of course the short transit time can be due to a grazing transit, that does not go across the disk center, but the transit curve looks flat bottom which argues against this. So, most likely this is an M dwarf star, based on the transit duration.

If you want to include the radius of the planet. The transit depth is 0.005 which implies an $R_{\mathrm{planet}} = 0.02 R_*$ (or $0.07\ R_\odot$, using $R_* = 0.28\ R_\odot$). The Keplerian velocity of the planet is (using Keplers law and $M_* = 0.4\ M_\odot$) is 114.3 km/s. Since the transit time duration (first contact to last contact) is given by $(2R_* + 2R_{\mathrm{planet}})/v$ (with $v = 114.3$ km/s), this corrects the stellar radius by $R_{\mathrm{planet}}/R_* = 2\%$. Thus we can safely ignore the radius of the planet.

Problem 4.3

The protostar luminosity caused by accretion of matter onto it, the so-called *accretion luminosity*, is obviously given by

$$L = \frac{G M_\star \dot{M}}{R_\star}$$

where $\dot{M}$ is the accretion rate, $M_\star$ and $R_\star$ the mass and the radius of the protostar. This equation is based on the assumption that the material transforms potential energy into kinetic energy during the approach and then into thermal energy and emission on impact. Assuming the solar mass and radius, and expressing the result in units of the "present-day" luminosity of the Sun, $L_\odot$, we have

$$\frac{L}{L_\odot} = \frac{GM_\odot \dot{M}}{L_\odot R_\odot} = \frac{7\cdot 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}\ \cdot 2\cdot 10^{30}\ \mathrm{kg}\cdot\left[10^{-5}\cdot 2\cdot 10^{30}\ \mathrm{kg}\ /(3\cdot 10^7\ \mathrm{s})\right]}{4\cdot 10^{26}\ \mathrm{W}\cdot 7\cdot 10^8\ \mathrm{m}} \approx 300$$

– a very high value!

The according Temperature at the solar surface can be estimated through the known relationship, valid for black-body radiation: $T \propto L^{1/4} R^{-1/2}$. For accretion onto the solar surface, we find

$$T \approx (L/L_\odot)^{1/4}\cdot T_\odot \approx 300^{1/4}\cdot 5800\ \mathrm{K} \approx 24000\ \mathrm{K}.$$

Assuming accretion/collapse onto a protostellar central cloud of larger extent, say 0.1 AU, we find

$$T \approx (L/L_\odot)^{1/4}\,(0.1\ \mathrm{AU}/R_\odot)^{-1/2}\cdot T_\odot \approx 5000\ \mathrm{K}.$$

Problem 4.4

Giant planets can form directly, when the disk is gravitationally unstable. The Toomre instability criterion reads

$$Q \equiv \frac{h}{r}\frac{M_*}{M_{\mathrm{disk}}} < 2.$$

Using the formula for the scale height, $h = c_\mathrm{s}/\Omega_\mathrm{K}$ or $h/r = c_\mathrm{s}/v_\mathrm{K}$, we rewrite the criterion as

$$\frac{c_\mathrm{s}}{v_\mathrm{K}} < 2\frac{M_{\mathrm{disk}}}{M_*}.$$

For the sound velocity we have

$$c_\mathrm{s} = \sqrt{\frac{kT}{\mu m_\mathrm{p}}}$$

yielding

$$\sqrt{\frac{kT}{\mu m_\mathrm{p}}} < 2\frac{M_{\mathrm{disk}}}{M_*} v_\mathrm{K}$$

or

$$T < 4\left(\frac{M_{\mathrm{disk}}}{M_*}\right)^2 \frac{\mu m_\mathrm{p}}{k} v_\mathrm{K}^2$$

Here, the Keplerian circular velocity is given by $v_\mathrm{K} = \sqrt{GM_*/r}$. Assuming $M_* = M_\odot$, we obtain $v_\mathrm{K} = 30\ \mathrm{km\ s}^{-1} = 3\cdot 10^4\ \mathrm{m\ s}^{-1}$ at $r = 1\mathrm{AU}$. With $\frac{M_{\mathrm{disk}}}{M_*} = 0.01$ and $\mu = 2$ (molecular hydrogen), the instability criterion is

$$T < 4\cdot 10^{-4}\frac{2\cdot 1.7\cdot 10^{-27}\ \mathrm{kg}}{1.4\cdot 10^{-23}\ \mathrm{J\ K}^{-1}}\cdot(3\cdot 10^4)^2 \sim 100\ \mathrm{K},$$

which is too cold! At Saturn's distance of 10AU, the required temperature is as low as 10K, which is absolutely unrealistic. But for $\frac{M_{\mathrm{disk}}}{M_*} \sim 0.1$ instead of 0.01 the critical temperature grows by two orders of magnitude, reaching reasonable values.