Physics of Planetary Systems — Exercises

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2009-05-07

Solutions to Set 2

Problem 2.1

Why is there such a tight correlation? By Keplers law we expect that the period should be correlated with the semi-major axis, if all stars have the same mass.

What about the outliers? The key to understanding this figure is Newtons form of Keplers law:

$$P^2 = \frac{4\pi^2 \left(a_{\rm star} + a_{\rm planet}\right)^3}{G\left(m_{\rm star} + m_{\rm planet}\right)} \tag{1}$$

There is a constant that depends on stellar mass (again assume $m_{\rm star} \gg m_{\rm planet}$). These outliers do not violate Keplers law, rather these stars have a very different mass than most stars on the curve.

What does this tell you? If the sample of planet hosting stars were unbiased, then we should expect exoplanets around the full range of stellar masses. If this were the case, we would see some correlation (the mass dependence is weak compared to period and semi-major axis) but we would expect a lot more scatter about the P–a relationship. The fact that this correlation is so tight tells you that most of the planet hosting stars have about the same mass – in this case 1 $M_\odot$. So this figure beautifully demonstrates that there is a strong bias in the sample, we only know about planets around stars in a very narrow range of stellar masses.

Problem 2.2

We count about $N = 75$ planets with masses in the range 1–2 $M_{\rm Jup}$. As an average (the number is small; it makes no difference if you take the average or the actual value) take $m_{\rm planet} \sin i = 1.5\ M_{\rm Jup}$. For these to have a true mass of at least 20 $M_{\rm Jup}$, the inclination of the orbit should be $i \leq \arcsin(1.5/20) = 4.3^\circ$. The probability that $i \leq 4.3^\circ$ is $P = (1 - \cos i) = 0.0028$. This is the probability that one of these exoplanets is at least a brown dwarf or more massive. The probability that all have masses greater than a brown dwarf of 20 $M_{\rm Jup}$ is just the product of all probabilities, $P_{\rm tot} = P^N = 5 \times 10^{-192}$, i.e. very small! In a more detailed study, one could calculate the probabilities individually for each planet and then multiply them. However, the conclusion would not change.

Vice versa, we can also calculate the probability that all of them *are* planets. The individual probabilities are given by $P' = 1 - P$ and the overall probability by $P'_{\rm tot} = P'^N = (1-P)^N = 0.81 = 81\%$. Assuming a limiting mass of 13 $M_{\rm Jup}$ instead of 20 $M_{\rm Jup}$, we obtain $P_{\rm tot} = 7 \times 10^{-164}$ and $P'_{\rm tot} = 60\%$.

Problem 2.3

The total energy of a spherical cloud is $E = K + U$ or, explicitly,

$$E = \frac{1}{2}\frac{3kT}{\mu m_{\rm p}}\mathscr{M} - \frac{3}{5}\frac{G\mathscr{M}^2}{R}.$$

Web: http://www.astro.uni-jena.de/~tloehne/planetensysteme/uebungen.html
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First, we have to find a mechanism that stopped the collapse. The rotation is not there, so that it can only be the pressure gradient. Then we are left with a standard problem: a sphere in hydrostatic equilibrium. The virial theorem must hold:

$$2K + U = 0.$$

Thus, from the virial theorem follows that the total energy

$$E = K + U = -K < 0!$$

In other words, the collapsing cloud must get rid of the initial energy, i.e. radiate it out, to reach an equilibrium. The energy is *not* conserved.

We can make estimates of the temperature. Note that the Sun will not be isothermal, so we can only make estimates of the mean temperature – in the spirit of the theory of stellar structure. Namely, from the virial theorem

$$K = -U/2$$

or

$$\frac{1}{2}\frac{3kT}{\mu m_\mathrm{p}}\mathscr{M} = \frac{3}{10}\frac{G\mathscr{M}^2}{R}$$

With $\mu = 2$ (molecular hydrogen), this results in

$$T = \frac{2G\mathscr{M}m_\mathrm{p}}{5kR}$$

or, numerically,

$$T \approx \frac{2\cdot 7\cdot 10^{-8}\cdot 2\cdot 10^{33}\cdot 2\cdot 10^{-24}}{5\cdot 1.4\cdot 10^{-16}\cdot 7\cdot 10^{10}}\ \mathrm{K} \approx 10^7\ \mathrm{K}.$$

Interestingly, this result is not very far from the temperature in the center of the real Sun: 15 million Kelvin.

Problem 2.4

Without any calculations one can surmise that stars of high luminosity are not very "friendly" to their disks: strong radiation pressure, photoevaporation, and other effects are able to erode the disk considerably or even destroy it.

Another thought: from "Introduction to Astronomy" and "Stellar Physics" lectures, you know that the "nuclear" lifetime t_nuc of a star with a high luminosity/mass is much shorter than that of a solar-type star. It can become comparable to, or even shorter than, the lifetime of a protoplanetary disk of $\sim 10^7$ years, leaving a planetary system not enough time to develop. Indeed, using the standard mass-luminosity relation for main-sequence stars, $L \propto \mathscr{M}^4$, one gets

$$t_\mathrm{nuc} \propto \frac{\mathscr{M}}{L} \propto \mathscr{M}^{-3},$$

implying that a 10 solar mass star has a 1000 times shorter lifetime, only $\sim 10^7$ yr.

Other reasonable arguments (as proposed by students):

- A planetary system bears the major fraction of a system's angular momentum. In a disk with smaller initial angular momentum, more mass can settle onto the star and less is ejected or available for the planet formation. The result may be a more massive star and a reduced probability for planets.

- Increasing the initial cloud mass (while keeping the specific angular momentum constant) leads to more massive stars and protoplanetary disks at the same time. The timescales for planet formation would decrease, the chances might increase.

Post Scriptum: Recent observations (J. A. Johnson et al., 2007) indicate an increasing probability for finding Jupiters around subgiants of increasing mass, i.e. around old A stars (with extended and cool atmospheres that allow application of the radial-velocity method) compared to sun-like stars. In addition, these newly found Hot Jupiters seem not as hot as (or as close in as) those detected around F, G, or K stars.

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