Physics of Planetary Systems — Exercises

Astrophysikalisches Institut und Universitätssternwarte Jena
Thüringer Landessternwarte Tautenburg

2009-04-29

Solutions to Set 1

Problem 1.1

1. *Use telluric lines in the Earth's atmosphere (e.g., O_2 at 6300 Å) as the reference*
   Advantages: Simple, inexpensive, can use any spectrograph can work.
   Disadvantages: Limited wavelength coverage, cannot control Earths atmosphere (temperature and pressure changes, winds).

2. *Simultaneous Thorium-Argon*
   Advantages: Large wavelength coverage, computationally simple
   Disadvantages: Contamination by Th-Ar, one cannot model instrumental profile, Th-Ar lamps change with time so long term stablity questionable. Spectrograph has to be specially built to accomodate fibers from Th-Ar and star. For the best stability spectrograph has to be thermally and mechanicaly stabilized.

3. *HF cell*
   Advantages: stable reference, no temperature, pressure, or wind shifts.
   Disadvantages: long path length is needed (1 m), HF (Hydrofluoric acid, Fluorwasserstoffsäure) is only present over a narrow wavelength range (100 Å), has to be refilled with each observing run, a dangerous gas to use.

4. *Iodine cell*
   Advantages: relatively safe to use, a short path length needed for cell (10 cm), relatively inexpensive, any existing high resolution spectrograph can use such a cell, cell is sealed permanently, can model the instrumental profile for better precision, excellent long term stability rather than stabilizing a whole spectrograph one only stabilizes a small cell.
   Disadvantages: Although it has a larger wavelength coverage (1000 Å) than HF it is still not as large as for Th-Ar technique. Contamination of spectral lines with iodine means that the spectral lines can be used only for RV work and little else, computationally intensive.

Problem 1.2

To calculate this, one needs to know about magnitudes, the fact that the RV error is rougly $(S/N) - 1$ (where S/N is the signal-to-noise ratio), and that S/N is proprtional to the square root of the number of detected photons. So we need to calculate the number of detected photons on the fainter star factoring in the fainter magnitude, plus the fact that one will expose longer: A magnitude difference of $\Delta m = 3.8$ corresponds to a brightness (photons) reduction by a factor of $2.512^{\Delta m} = 2.512^{3.8} = 33.11$. This means in 22 seconds on a 15.8 mag star, Keck HiRes will detect 1/33.1 the number of photons (i.e. $1/\sqrt{33.1} = 1/5.75$ times the signal to noise ratio since $S/N \propto \sqrt{\text{detected photons}}$). But by increasing the exposure time, one detects more photons by a factor $3600/22 = 164$. So compared to the $m_\text{v} = 12$ with a 22 s exposure, on a $m_\text{v} = 15.8$ mag star Keck HiRes will detect in one hour $(1/33.1) \times 164 = 5$ times the number of photons. Since RV error is inversely proportional to S/N and the S/N is proportional to the square root of the detected photons, one will have an RV error $\frac{2}{5} = 0.4$ m/s. Note: at these faint magnitudes systematic errors (e.g. characteristics of the detector) become an important contribution, so the error may be much larger than for photon statistics.

Problem 1.3

Recall the mass function given in the lecture:

$$f(m) = \frac{\left(m_{\text{planet}} \sin i\right)^3}{(m_{\text{star}} + m_{\text{planet}})^2} = \frac{PK^3 \left(1 - e^2\right)^{3/2}}{2\pi G}. \tag{1}$$

Of course you have to know the stellar mass, which was neglected to give, but lets assume that the masses of the stars are the same as well as their orbital inclinations. Let m_c and m_e (both $\ll m_{\text{star}}$) be the masses of the circular and eccentric planets, and f_c and f_e their respective mass functions. Taking the ratio of the mass functions and ignoring the stellar masses and $\sin i$ which we take to be the same for both stars, we have

$$\frac{f_e}{f_c} = \left(\frac{m_e}{m_c}\right)^3 = \frac{\left(1 - e^2\right)^{3/2}}{1} \quad = \quad 0.08,$$

and thus $m_e = 0.43 m_c$. So, the planet in the more eccentric orbit has about 40% of the mass of the one in the circular orbit.

Problem 1.4

From the virial theorem, the condition for the Jeans radius or mass is $K = |U|/2$, where

$$K = \frac{1}{2}\mathscr{M}v^2 = \frac{1}{2}\frac{3kT}{\mu m_p}\mathscr{M} \qquad \text{and} \qquad |U| = \frac{3}{5}\frac{G\mathscr{M}^2}{R},$$

so that

$$\frac{kT}{\mu m_p}\mathscr{M} = \frac{1}{5}\frac{G\mathscr{M}^2}{R}, \tag{2}$$

giving the Jeans radius

$$R_J = \frac{1}{5}\mu m_p \frac{G\mathscr{M}}{kT},$$

which contains an additional factor 1/5 (and μ, of course).

Again introducing the volume number density n and thus the volume mass density $n\mu m_p$, we now re-derive the Jeans mass:

$$\mathscr{M}_J \sim n\mu m_p \cdot \frac{4}{3}\pi R_J^3 \qquad \text{or} \qquad R_J \sim \left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{\mathscr{M}_J}{n\mu m_p}\right)^{1/3}$$

Substituting this into equation (2), one yields

$$\frac{kT}{\mu m_p} = \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} G\mathscr{M}_J^{2/3}(n\mu m_p)^{1/3}$$

or

$$kT = \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} G\mathscr{M}_J^{2/3} n^{1/3}(\mu m_p)^{4/3}$$

or

$$\mathscr{M}_J = 5^{3/2}\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{kT}{Gn^{1/3}(\mu m_p)^{4/3}}\right)^{3/2}$$

or

$$\mathscr{M}_J = 5^{3/2}\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{k}{G}\right)^{3/2}\frac{1}{(\mu m_p)^2}\frac{T^{3/2}}{n^{1/2}},$$

where the numerical prefactor is 5.46.

Problem 1.5

Taking into account that the angular momentum is conserved, the rotation period of the protosun would be as short as

$$P = \frac{2\pi}{\Omega} = \frac{2\pi}{\frac{L}{\mathscr{M}R^2}} = 2\pi \frac{R^2}{L/\mathscr{M}} \sim 6 \cdot \frac{(7 \cdot 10^{10})^2}{10^{21}} \sim 30 \text{ s} \sim 0.5 \text{ min}.$$

On the other hand, the minimum rotation period $P_{\rm min}$, at which the protosun would not be broken by the centrifugal force, is the orbital period of a Keplerian orbit with radius $R_\odot$. Were the orbital radius equal to 1 AU, we would have $P_{\rm min} = 1$ year. For the orbital radius of $R_\odot = 1/200$ AU, the minimum period according to Kepler's 3rd law is

$$P_{\rm min} = 1 \text{ year} \cdot \left(\frac{1}{200}\right)^{3/2} \sim \frac{1 \text{ year}}{200 \cdot 15} \sim \frac{400 \text{ days} \cdot 1440 \ \frac{\text{minutes}}{\text{day}}}{200 \cdot 15} \sim 200 \text{ min},$$

which is much longer than the period derived purely from angular momentum conservation. Thus, only a small fraction of the cloud's original total angular moment can be transferred to the star itself.