Mathematische Methoden der Physik I - Klausur
FSU Jena - WS 06/07
- Lösungen -

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Aufgabe 1

$$\frac{dr}{d\phi} + r\tan(\phi) = 0 \Rightarrow \int \frac{dr}{r} = -\int \tan(\phi)d\phi \Rightarrow \ln(r) = \ln(\cos(\phi)) + C,\ C : const\ \ \Rightarrow r = A \cdot \cos(\phi),\ A : const$$

Spezielle Lösung : $r = -2\cos(\phi)$.

Aufgabe 2

Homogene:

$$t\frac{ds}{dt} - 2s = 0 \Rightarrow \int \frac{ds}{s} = \int \frac{2dt}{t} \Rightarrow \ln(s) = 2\ln(t) + C,\ C : const \Rightarrow s = A \cdot t^2,\ A : const$$

Variation der Konstanten:

$$s = u(t)t^2 \Rightarrow s' = u't^2 + 2ut \Rightarrow ts' - 2s = u't^3 + 2ut^2 - 2s = u't^3 = t^3\ln(t)$$

$$\Rightarrow u = \int \ln(t)dt = t\ln(t) - \int \frac{tdt}{t} = t(\ln(t) - 1) + C,\ C : const\ \ \Rightarrow s = t^3\left[ln(t) - 1\right] + Ct^2$$

Aufgabe 3

a) Die erste ist Exakt. Die zweite nicht.

b)

$$A := 3x^2e^y \stackrel{!}{=} \frac{\partial U}{\partial x},\ B := x^3e^y - 1 \stackrel{!}{=} \frac{\partial U}{\partial y}$$

$$\Rightarrow U = \int Adx = x^3e^y + f(y) \Rightarrow \frac{\partial U}{\partial y} = x^3e^y + \frac{df}{dy} \stackrel{!}{=} B = x^3e^y - 1$$

$$\Rightarrow \frac{df}{dy} = -1 \Rightarrow f = -y + C,\ C : const$$

$$\Rightarrow U = x^3e^y - y + C \stackrel{!}{=} const \Rightarrow x^3e^y - y = R,\ R : const$$

c)

$$A := e^{2x} - y^2,\ B := y,\ Gesucht : \lambda = \lambda(x) : \frac{\partial(A\lambda)}{\partial y} = \frac{\partial(B\lambda)}{\partial x}$$

$$\Rightarrow \frac{\partial A}{\partial y}\lambda = \frac{\partial B}{\partial x}\lambda + B\frac{d\lambda}{dx} \Rightarrow -2y\lambda = y\frac{d\lambda}{dx} \Rightarrow \int \frac{d\lambda}{\lambda} = -2\int dx$$

$$\Rightarrow \lambda = e^{-2x}$$

Aufgabe 4

Homogene:

$$\ddot{x} + 2k\dot{x} + 2k^2x = 0,\ Ansatz : x = e^{\lambda x} \Rightarrow \lambda^2 + 2k\lambda + 2k^2 = 0 \Rightarrow \lambda = -k \pm ki$$

$$\Rightarrow x_h = C_1e^{-k(1-i)} + C_2e^{-k(1+i)},\ C_1, C_2 : const$$

Inhomogene:

$$Ansatz : x = \alpha\sin(kt) + \beta\cos(kt) \Rightarrow \dot{x} = k\alpha\cos(kt) - k\beta\sin(kt),\ \ddot{x} = -k^2\alpha\sin(kt) - k^2\beta\cos(kt)$$

$$\Rightarrow \ddot{x} + 2k\dot{x} + 2k^2x = -k^2\alpha\sin(kt) - k^2\beta\cos(kt) + 2k^2\alpha\cos(kt) - 2k^2\beta\sin(kt) + 2k^2\alpha\sin(kt) + 2k^2\beta\cos(kt) \stackrel{!}{=} 5k^2\sin(kt)$$

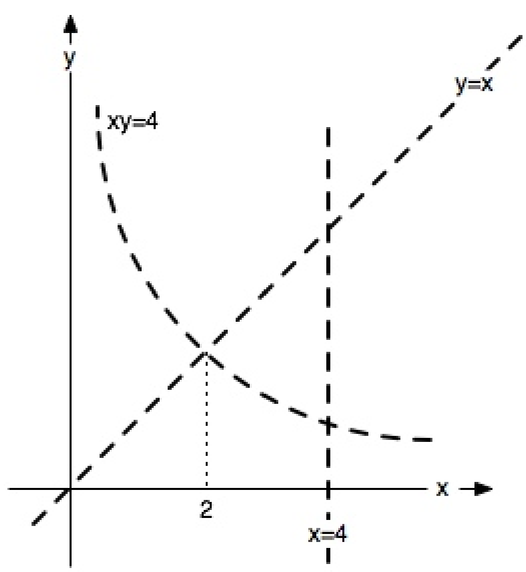
$$Komponentenvergleich : k^2\alpha - 2k^2\beta = 5k^2 \wedge k^2\beta + 2k^2\alpha = 0 \ \Rightarrow\ \alpha = 1 \wedge \beta = -2$$

$$\Rightarrow x = C_1e^{-k(1-i)} + C_2e^{-k(1+i)} + \sin(kt) - 2\cos(kt)$$

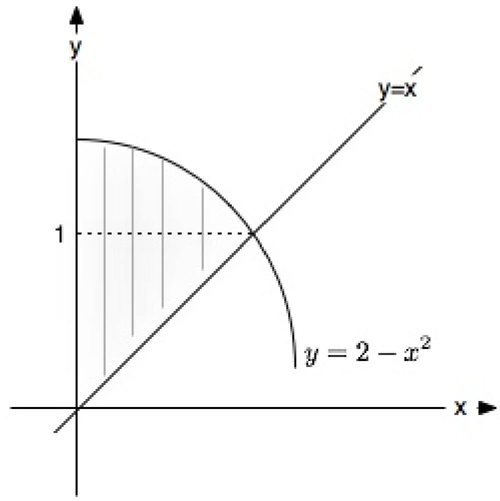
Aufgabe 5

a)

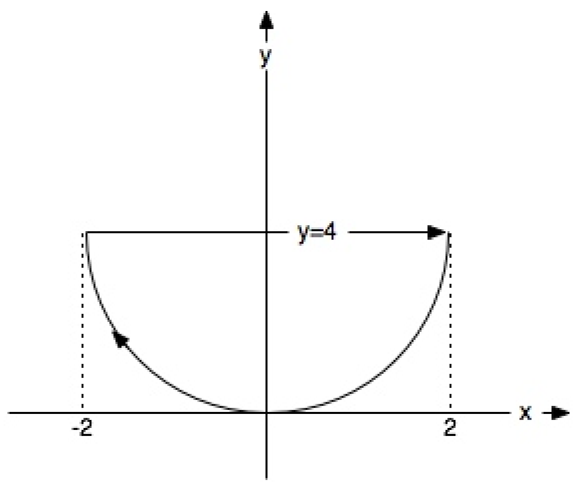
$$A = \int_2^4 \int_{y_1}^{y_2} dydx,\ wobei : y_1 = \frac{4}{x}\ \wedge\ y_2 = x \Rightarrow A = \int_2^4 \left(x - \frac{4}{x}\right) dx = \left[\frac{x^2}{2} - 4\ln(x)\right]_2^4 = 6 - 4\ln(2)$$



b) Gebiet:



Aufgabe 6



$$W = \int_C [ydx + (2x + y))dy] = \int_2^{-2} \left[x^2 + 2x(2x + x^2)\right] dx + \int_{-2}^{2} 4dx = -\frac{80}{3} + 16 = -\frac{32}{3}$$

Aufgabe 7

$$P := y^2,\ Q := (x + y)^2$$

$$W = \int_C \left[y^2 dx + (x + y)^2 dy\right] = \int\int_A \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dxdy = \int\int_A 2xdxdy$$

$$= 2\int_0^a (a - x)xdx = \frac{2a^3}{3}$$

Aufgabe 8

a)

$$P := yz,\ Q := xz,\ R := xy$$

$$U = \int_{x_0}^{x} P(\xi, y_0, z_0) d\xi + \int_{y_0}^{y} Q(x, \xi, z_0) d\xi + \int_{z_0}^{z} R(x, y, \xi) d\xi = \int_{x_0}^{x} y_0 z_0 d\xi + \int_{y_0}^{y} x z_0 d\xi + \int_{z_0}^{z} xy d\xi$$

$$= xy_0z_0 - x_0y_0z_0 + xyz_0 - xy_0z_0 + xyz - xyz_0 = xyz - x_0y_0z_0 = xyz + U_0,\ U_0 := -x_0y_0z_0 : const$$

b)

$$rot\vec{\Phi} = (0,0,0) \Rightarrow \forall\ Geschlossene\ Wege\ C : \oint_C \vec{\Phi} d\vec{r} = \int_A rot\vec{\Phi} d\vec{f} = 0\ \Rightarrow\ \vec{\Phi}\ Konservativ \quad \square$$