Magnetohydrodynamik
FSU Jena - SS 2011
Serie 04 - Lösungen

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Aufgabe 06

Es sei

$$R_{ik} := \eta \left(\partial_k \mathrm{v}^i + \partial_i \mathrm{v}^k\right) - \left(\frac{2}{3}\eta - \zeta\right) \delta_{ik} \operatorname{div} \mathbf{v}. \tag{0.1}$$

Dann ist

$$\begin{aligned}
\partial_k R_{ik} &= \eta \left(\partial_k \partial_k \mathrm{v}^i + \partial_k \partial_i \mathrm{v}^k\right) - \left(\frac{2}{3}\eta - \zeta\right) \delta_{ik} \partial_k \operatorname{div} \mathbf{v} \\
&= \eta \Delta \mathrm{v}^i + \eta \partial_i \operatorname{div} \mathbf{v} - \left(\frac{2}{3}\eta - \zeta\right) \partial_i \operatorname{div} \mathbf{v} \\
&= \eta \Delta \mathrm{v}^i + \left(\frac{\eta}{3} + \zeta\right) \partial_i \operatorname{div} \mathbf{v} = k_i^{(\mathrm{Reib})}.
\end{aligned} \tag{0.2}$$

Aufgabe 07

Es bezeichne

$$V_{ik} := \frac{1}{2} \left(\partial_k \mathrm{v}^i + \partial_i \mathrm{v}^k\right). \tag{0.3}$$

Dann lässt sich schreiben

$$\begin{aligned}
R_{ik} \frac{\partial \mathrm{v}^i}{\partial x^k} &\overset{(0.1)}{=} \eta \left[(\partial_k \mathrm{v}^i)(\partial_k \mathrm{v}^i) + (\partial_k \mathrm{v}^i)(\partial_i \mathrm{v}^k)\right] - \left(\frac{2}{3}\eta - \zeta\right) \underbrace{(\delta_{ik} \partial_k \mathrm{v}^i)}_{\operatorname{div} \mathbf{v}} \operatorname{div} \mathbf{v} \\
&= \frac{1}{2}\eta \left[(\partial_k \mathrm{v}^i)(\partial_k \mathrm{v}^i) + (\partial_i \mathrm{v}^k)(\partial_i \mathrm{v}^k) + 2(\partial_k \mathrm{v}^i)(\partial_i \mathrm{v}^k)\right] - \left(\frac{2}{3}\eta - \zeta\right) (\operatorname{div} \mathbf{v})^2 \\
&= \frac{1}{2}\eta \left(\partial_k \mathrm{v}^i + \partial_i \mathrm{v}^k\right) \left(\partial_k \mathrm{v}^i + \partial_i \mathrm{v}^k\right) - \left(\frac{2}{3}\eta - \zeta\right) (\operatorname{div} \mathbf{v})^2 \\
&= 2\eta V_{ik} V_{ik} - \left(\frac{2}{3}\eta - \zeta\right) (\operatorname{div} \mathbf{v})^2.
\end{aligned} \tag{0.4}$$

Aufgabe 08

Durch direktes Ausrechnen zeigt sich

$$\begin{aligned}
\rho \frac{d}{dt} \frac{A}{\rho} - \operatorname{div}(A\mathbf{v}) &= \rho \partial_t \frac{A}{\rho} + \rho \mathbf{v} \operatorname{grad} \frac{A}{\rho} - \mathbf{v} \operatorname{grad} A - A \operatorname{div} \mathbf{v} \\
&= \left[\partial_t A - \frac{A}{\rho} \partial_t \rho\right] + \mathbf{v} \left[\cancel{\operatorname{grad} A} - \frac{A}{\rho} \operatorname{grad} \rho\right] - \cancel{\mathbf{v} \operatorname{grad} A} - A \operatorname{div} \mathbf{v} \\
&= \partial_t A - \frac{A}{\rho} \left[\partial_t \rho + \mathbf{v} \operatorname{grad} \rho + \rho \operatorname{div} \mathbf{v}\right] = \partial_t A - \frac{A}{\rho} \underbrace{\left[\partial_t \rho + \operatorname{div}(\rho \mathbf{v})\right]}_{\substack{0 \\ \text{(Kontinuitätsgleichung)}}} \\
&= \partial_t A.
\end{aligned} \tag{0.5}$$