High-Intensity / Relativistic Optics

Problem set 3—mean temperature, Debye length, plasma frequency

1. **Exercise:** In a strictly steady state situation, both the ions and the electrons obey a Boltzmann distribution

$$n_{\rm j} = n_0 \exp\left\{\frac{-q_{\rm j}\phi}{{\rm k}_{\rm B}T_{\rm j}}\right\}\,.$$

For the case of an infinite, transparent grid charged to a potential ϕ , show that the shielding distance is then given approximately by

$$\frac{1}{\lambda_{\rm D}^2} = \frac{n_0 {\rm e}^2}{\epsilon_0} \left(\frac{1}{{\rm k_{\rm B}} T_{\rm e}} + \frac{1}{{\rm k_{\rm B}} T_{\rm i}} \right) \,. \label{eq:Lagrangian}$$

Show that $\lambda_{\rm D}$ is determined by the temperature of the colder species.

2. Exercise: The plates of a rectangular capacitor are at a potential difference of $\pm \phi_0$ and at a distance of 2R. The capacitor is filled with a plasma which has a Debye length of λ_D . What is the potential and the electric fields between the capacitor plates?

Discuss the limits of $R \gg \lambda_{\rm D}$ and $R \ll \lambda_{\rm D}$.

3. Exercise: Improve the derivation of the plasma frequency $\omega_{\rm P}$ by taking into account the mass of the ions. Assume that the ions in the ion layer all move together.