GR: Problem Set #12

1. (Carroll, Problem 7.4.) Show that the Lorenz gauge condition, $\partial_\mu \bar{h}^{\mu\nu} = 0$ is equivalent to the **harmonic gauge** condition. This gauge is defined by

$$\Box x^\mu = 0,$$

   where each coordinate x^μ is thought of as a scalar function on spacetime. (Any function satisfying $\Box f = 0$ is known as an "harmonic function".)

2. (Carroll, Problem 7.6.) Two objects of mass M have a head-on collision at event $(0, 0, 0, 0)$. In the distant past, $t \to -\infty$, the masses started at $x \to \pm\infty$ with zero velocity.

   (a) Using Newtonian theory, show that $x(t) = \pm(9Mt^2/8)^{1/3}$.

   (b) For what separations is the Newtonian approximation reasonable?

   (c) Calculate $h^{TT}_{xx}(t)$ at $(x, y, z) = (0, R, 0)$.

3. In the linearized gravitational theory, show that the equations of motion for matter $T^{\mu\nu}{}_{;\nu} = 0$ are inconsistent with the field equations for the metric perturbation. Show that this inconsistency is of second order in the metric perturbation, hence negligible to first order.

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