General Theory of Relativity
FSU Jena - WS 2009/2010
Problem set 05

December 15, 2009

Problem 01 (Carroll, Problem 2.4)

Verify the claims made about the commutator of two vectors fields (linearity, Leibniz, component formula, transformation as a vector field)

$$[X,Y](af+bg) = a\,[X,Y](f) + b\,[X,Y](g)$$
$$[X,Y](fg) = f\,[X,Y](g) + g\,[X,Y](f)$$
$$[X,Y]^\mu = X^\nu\partial_\nu Y^\mu - Y^\nu\partial_\nu X^\mu$$

where a and b are real numbers and f and g are functions.

Problem 02 (Carroll, Problem 2.7)

Prolate spheroidal coordinates can be used to simplify the Kepler problem in celestial mechanics. They are related to the usual Cartesian coordinates (x, y, z) of Euclidean three-space by

$$x = \sinh\chi\sin\vartheta\cos\varphi$$
$$y = \sinh\chi\sin\vartheta\sin\varphi$$
$$z = \cosh\chi\cos\vartheta$$

Restrict your attention to the plane $y = 0$ and answer the following questions.

(a) What is the coordinate transformation matrix $\frac{\partial x^\mu}{\partial x^{\nu'}}$ relating (x, y) to (χ, ϑ)?

(b) What does the line element ds^2 look like in prolate spheroidal coordinates?

Problem 03 (Carroll, Problem 2.8)

Verify the following statement: For exterior derivative of a product of a p-form ω and q-form η we have

$$d(\omega\wedge\eta) = (d\omega)\wedge\eta + (-1)^p\omega\wedge(d\eta)$$