# General Theory of Relativity
# FSU Jena - WS 2009/2010
# Problem set 02

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## Problem 01 (Carroll, Problem 1.3)

Three events, $A$, $B$ and $C$, are seen by observer $O$ to occur in order $ABC$. Another observer, $\widetilde{O}$, sees the events to occur in the order $CBA$. Is it possible that a third observer sees the events in the order $ACB$? Support your conclusion by drawing a spacetime diagram.

## Problem 02 (Carroll, Problem 1.10)

Using the tensor transformation law applied to $F_{\mu\nu}$, show how the electric and magnetic 3-vectors **E** and **B** transform under

(a) a rotation about the $y$-axis

(b) a boost along the $z$-axis.

## Problem 03

Two equivalent inertial frames $S$ and $S'$ are such that $S'$ moves in the positive $x$ direction with speed $v$ as seen from $S$. The spatial coordinate axes in $S'$ are parallel to those in $S$ and the two origins are coincident at time $t = t' = 0$.

(a) Show that the isotropy and homogeneity of space-time and equivalence of different inertial frames (first postulate of relativity) require that the most general transformation between the space-time coordinates $(t, x, y, z)$ and $(t', x', y', z')$ is the linear transformation

$$x' = f(v^2)x - vf(v^2)t \ ; \ t' = g(v^2)t - vh(v^2)x \ ; \ y' = y \ ; \ z' = z$$

and the inverse

$$x = f(v^2)x' + vf(v^2)t' \ ; \ t = g(v^2)t' + vh(v^2)x' \ ; \ y = y' \ ; \ z = z'$$

where $f, g$ and $h$ are functions of $v^2$, the structures of the $x'$ and $x$ equations are determined by the definition of the inertial frames in relative motion, and the signs in the inverse equation are a reflection of the reversal of roles of the two frames.

(b) Show that consistency of the initial transformation and its inverse require

$$f = g \qquad \text{and} \qquad f^2 - v^2 fh = 1$$

(c) If a physical entity has speed $u'$ parallel to the $x'$ axis in $S'$, show that its speed $u$ parallel to the $x$-axis in $S$ is

$$u = \frac{u' + v}{1 + vu'(h/f)}$$

Using the second postulate of relativity (universal limiting speed $c$), show that $h = f/c^2$ is required and that the Lorentz transformation of the coordinates results. The universal limiting speed $c$ is to be determined from experiment.