General Theory of Relativity
FSU Jena - WS 2009/2010
Problem set 01

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Problem 01 (Carroll, Problem 1.4)

Projection effects can trick you into thinking that an astrophysical object is moving *superliminary*. Consider a quasar that ejects gas with speed v at an angle ϑ with respect to the line-of-sight of the observer. Projected onto the sky, the gas appears to travel perpendicular to the line of sight with angular speed $v_{\rm app}/D$, where D is the distance to the quasar and $v_{\rm app}$ is the apparent speed. Derive an expression for $v_{\rm app}$ in terms of v and ϑ. Show that, for appropriate values of v and ϑ, $v_{\rm app}$ can be greater than 1.

Problem 02 (Carroll, Problem 1.5)

Particle physicists are so used to setting $c = 1$ that they measure mass in units of energy. In particular, they tend to use electron volts (1 eV $= 1.6 \times 10^{-12}$ erg = 1.8×10^{-33} g), or, more commonly, keV, MeV and GeV (10^3eV, 10^6 eV and 10^9eV, respectively). The muon has been measured to have a mass of 0.106 GeV, and a rest frame lifetime of 2.19×10^{-6} s. Imagine that such a muon is moving in a circular storage ring of a particle accelerator, 1 kilometer in diameter, such that the muon's total energy is 1000 GeV. How long would it appear to live from the experimenter's point of view? How many radians would it travel around the ring?

Problem 03 (Carroll, Problem 1.7)[1]

Imagine we have a tensor $X^{\mu\nu}$ and a vector V^μ, with components

$$X^{\mu\nu} = \begin{pmatrix} 6 & 0 & 1 & 0 \\ -1 & 0 & -12 & 2 \\ 1 & 6 & 0 & 0 \\ -8 & 1 & 1 & -6 \end{pmatrix} \quad , \quad V^\mu = (-1, -2, 0, 2)$$

Find the components of

(a) $X^\mu{}_\nu$

(b) $X_\mu{}^\nu$

(c) $X^{(\mu\nu)}$

(d) $X_{[\mu\nu]}$

(e) $X^\lambda{}_\lambda$

(f) $V^\mu V_\mu$

(g) $V_\mu X^{\mu\nu}$

[1] Different components