Gravitationstheorie I
FSU Jena - WS 2009/2010
Klausur

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Problem 01

Prove that the Einstein equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ in a vacuum spacetime can be rewritten as $R_{\mu\nu} = 0$.

Problem 02

Show that metric compatibility of a covariant derivative operator ∇_μ implies

$$\Gamma^\sigma_{\mu\nu} = \frac{g^{\sigma\rho}}{2} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right)$$

(hint: compute a convenient linear superposition of the three cyclic permutations of $\nabla_\rho g_{\mu\nu}$).

Problem 03

Consider the *Schwarzschild metric* in 2+1 dimensions (not in 3+1),

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\phi^2 \quad (0.1)$$

(a) Is ds^2 invariant under coordinate transformations? Define the Killing vectors associated with rotations and time translations.

(b) Write the conserved quantity connected with each of the Killing vector and explain what they represent.

(c) Using $u^\mu u_\mu = -1$, find an expression equivalent to the equation

$$E = \frac{\dot{r}^2}{2} + V(r) + \frac{L^2}{2r^2} \quad (0.2)$$

for a particle moving under central force in classical mechanics.

Problem 04

Gravitational waves can be detected by monitoring the distance between two free flying masses. If one of the masses is equipped with a laser and accurate clock, and the other with a good mirror, the distance between the masses can be measured by timing how long it takes for a pulse of laser light to make the round-trip journey. How would you orient your detector to register the largest response from a plane wave of the form

$$ds^2 = -dt^2 + [1 + A\cos(\omega(t-z))]\, dx^2 + [1 - A\cos(\omega(t-z))]\, dy^2 + dz^2$$

? If the masses have a mean separation L, what is the largest change in the arrival time of the pulses caused by the wave? What frequencies would go undetected?

Problem 05

A space monkey is happily orbiting around a Schwarzschild black hole in a circular geodesic orbit. An evil snake, far from the black hole, tries to send the monkey to its death inside the black hole by dropping a carefully timed coconut radially toward the black hole, knowing that the monkey cant resist catching the falling coconut. Given the monkey's mass and initial orbital radius and the mass of the coconut, explain how you would go about solving the problem (**but do not do the calculation**). What are the possible fates of our intrepid space monkey? Argue with a qualitative plot of the effective potential for a geodesic in Schwarzschild.