JÜLICH FORSCHUNGSZENTRUM

Simulation of Drift-Kinetic Dynamics of Charged Particles in Magnetized Plasmas

The Guiding Center Approach

September 29, 2009 | Stilianos Louca

Presentation overview

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Part I: Introduction

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Introduction to Plasmas

Definition of plasmas

Systems on which thermal energies exceed ionization energies of atoms.
→ Assemblage of unbound electron & ions.

Plasma dynamics characterized by:

- High coupling due to internal electromagnetic fields.
- No bound states → complex, vigorous dynamics.

Particle Kinetics in Plasmas

- Single particle movement governed by Lorentz-Force:

$$\ddot{\mathbf{r}} = \frac{q}{m}\mathbf{E} + \frac{q}{m}\dot{\mathbf{r}} \times \mathbf{B} \tag{1}$$

  with position $\mathbf{r}$, charge q, mass m, electric field $\mathbf{E}$ and magnetic flux density $\mathbf{B}$.
- Fusion plasma confinement: requires strong, external magnetic fields $\mathbf{B} \sim 1$ T.
- Particle motion characterized by high-frequency gyrations.
- Other interactions: particle collisions, radiation.

Simulating Fusion Plasmas with PEPC

PEPC

Pretty **E**fficient **P**arallel **C**oulomb-solver

> *Parallel tree-code for rapid computation of long-range Coulomb forces in N-particle systems.*

[PEPC]

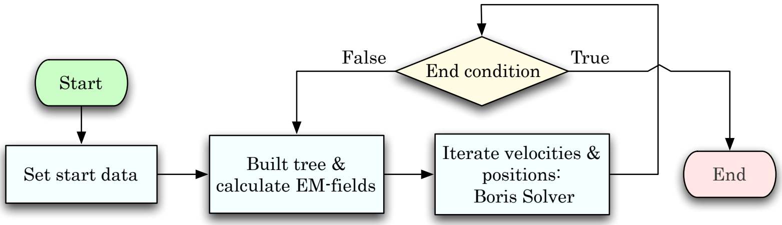

Figure: Basic PEPC flow-chart

Gyrations in Magnetic Fields

- **B**-field $\rightarrow$ charged particles circulate around field axis.
- $\curvearrowright$ Particles *approximately* follow **B**-field lines on spiral orbits.

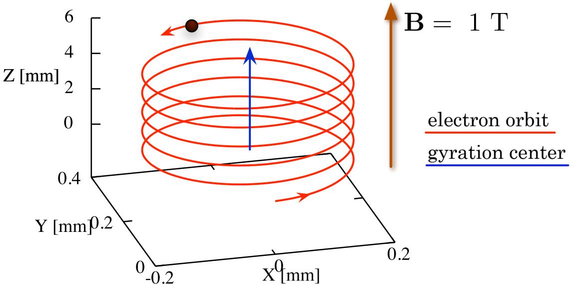

Figure: Electron gyration in homogenic **B**-field.

Scales in Fusion Plasma Simulations

Electron gyration radius	ρ_g	~ 0.1 mm
Electron gyration period	T_g	$\sim 10^{-12}$ s
Ion gyration period		$\sim 10^{-9}$ s

- Interesting phenomena at time scales ~ 1 s.
- Problem: Boris Solver requires time-step

$$\delta t \lesssim T_g \approx 10^{-13} \text{ s} \tag{2}$$

  for electron gyration.[Parker, Birdsall]
- Idea: Replace exact electron position $\mathbf{r}$ with gyration center $\mathbf{R}$.

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Part II: Guiding Center Motion

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Assumptions & scaling parameter

1. Only weak field variation within gyration scales.
2. *Interesting* particle interaction scales $\gg$ gyration scales.
3. Radiation neglectable in particle motion.
4. Expansion parameter:

$$\varepsilon := \rho_{\mathrm{g}} \cdot \frac{\|\nabla \mathbf{B}\|}{B} \overset{!}{\ll} 1 \tag{3}$$

with local gyration radius ρ_{g}.[Northrop]
$\rightarrow$ Ratio of gyration scales to field-variation scales.

Guiding Center: Definition

- Abstract: Geometrical center of local gyration.
- Exact [Northrop]:

$$\mathbf{R} := \mathbf{r} - \frac{\mathbf{B}}{\omega_{\mathrm{g}} B} \times \left(\dot{\mathbf{r}} - \frac{\mathbf{E} \times \mathbf{B}}{B^2} \right) \tag{4}$$

with local gyration frequency ω_{g}, particle position $\mathbf{r}$ & velocity $\dot{\mathbf{r}}$.

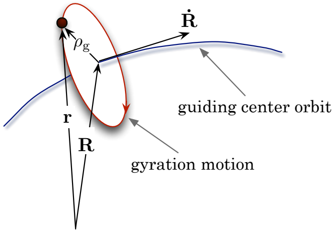


Figure: Guiding center of charged particle orbit.

Guiding Center: Equation of Motion

- Equation of motion for guiding center:

$$\boxed{\ddot{\mathbf{R}} = \frac{q}{m}\left[\mathbf{E} - \frac{\mu}{q}\nabla B\right] + \frac{q}{m}\dot{\mathbf{R}} \times \mathbf{B} + \mathcal{O}(\varepsilon)} \tag{5}$$

  with the *magnetic moment*

$$\mu := \frac{m}{2}\frac{\dot{\mathbf{r}}_\perp^2}{B} \tag{6}$$

  and particle velocity $\dot{\mathbf{r}}_\perp$ perpendicular to $\mathbf{B}$.[Northrop]

- Same structure as Lorentz-force → same kind of motion in first order.

Guiding Center: Perpendicular Drifts

Projection of the EOM perpendicular to **B** leads to:

$$\dot{\mathbf{R}}_\perp = \underbrace{\frac{\mathbf{E}\times\mathbf{B}}{B^2}}_{\mathbf{E}\times\mathbf{B}-\text{drift}} + \underbrace{\frac{\mu}{qB}\mathbf{b}\times\nabla B}_{\text{magnetic drift}} - \underbrace{\frac{m}{qB}\ddot{\mathbf{R}}\times\mathbf{b}}_{\text{acceleration drift}} + \mathcal{O}(\varepsilon^2) \tag{7}$$

with velocity $\dot{\mathbf{R}}_\perp$ perpendicular to **B**. ↷

- Guiding center drifting perpendicular to field lines.
- Drifts caused by variations of gyro-radius or gyration speed during gyration period.

$\mathbf{E} \times \mathbf{B}$-Drift

- Caused by variation of gyration speed due to **E**-field.
- Always perpendicular to **E** & **B**-fields.

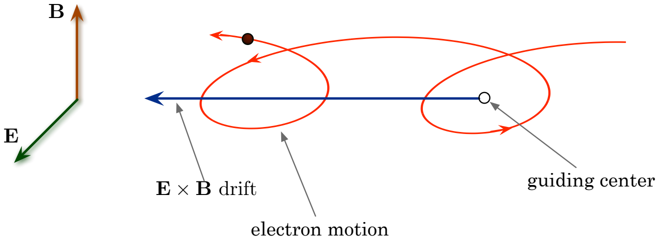


Figure: $\mathbf{E} \times \mathbf{B}$-drift of electron guiding center.

Guiding Center: Start Velocity

- Guiding center start *velocity*:

$$\dot{\mathbf{R}}(0) = \dot{\mathbf{r}}_{\parallel}(0) \; + \; \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \mathcal{O}(\varepsilon) \tag{8}$$

with particle velocity $\dot{\mathbf{r}}_{\parallel}$ parallel to $\mathbf{B}$.

- In fusion plasmas:

$$\dot{\mathbf{r}} \sim 10^7 \text{ m} \cdot \text{s}^{-1} \quad \gg \quad \frac{\mathbf{E} \times \mathbf{B}}{\mathrm{B}^2} \sim 1 \text{ m} \cdot \text{s}^{-1} \tag{9}$$

$\rightarrow$ leads to much smaller *gyration radius* of guiding center.

The Magnetic Moment μ

- Interpretation: Magnetic moment of charge orbiting on circle.
- Connected to *mean kinetic energy*:

$$\mathcal{E}_{\text{kin}} = \frac{m}{2}\dot{\mathbf{R}}^2 + \mu \cdot B \tag{10}$$

with

$$\frac{d\mathcal{E}_{\text{kin}}}{dt} = q\langle \dot{\mathbf{R}}, \mathbf{E} \rangle + \mu \frac{\partial B}{\partial t} + \mathcal{O}(\varepsilon^2) \tag{11}$$

- **Problem:** μ no longer available, since exact particle velocity unknown.
- **Solution:** Integrate μ ($\leftrightarrow\ \mathcal{E}_{\text{kin}}$) along with $\mathbf{R}, \dot{\mathbf{R}}$ using eq. (11).

Guiding Center: Numerical Integration

- Similarity to original EOM $\rightarrow$ usage of existing integration scheme.
- $\mathbf{E}$-field *modified* by ∇B-term.
- Leapfrog integration scheme:

$$\left(\mathbf{R}^n, \dot{\mathbf{R}}^{n-\frac{1}{2}}\right) \xrightarrow{\substack{\text{Boris}\\\text{Solver}}} \left(\mathbf{R}^{n+1}, \dot{\mathbf{R}}^{n+\frac{1}{2}}\right) \tag{12}$$

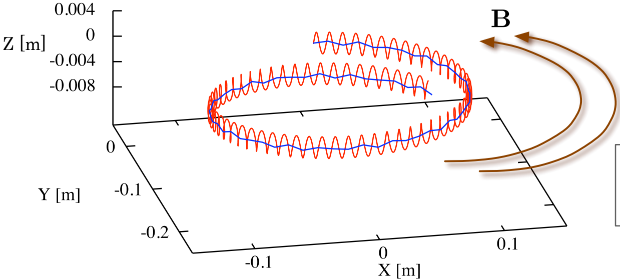
$$\mathcal{E}_{\text{kin}}^{n+\frac{1}{2}} := \mathcal{E}_{\text{kin}}^{n-\frac{1}{2}} + \delta t \cdot q \cdot \left\langle \mathbf{E}^{n+1}, \dot{\mathbf{R}}^n \right\rangle + \delta t \cdot \frac{\mu^{n-\frac{1}{2}}}{q} \cdot \nabla B^{n+1} \tag{13}$$

Fields

- $\mathbf{E}^n$ $\rightarrow$ calculated using parallel tree-code from positions $\mathbf{R}^n$.[PEPC]
- $\mathbf{B}^n$ $\rightarrow$ provided as external field at positions $\mathbf{R}^n$.

Example Simulation

Electron & Guiding Center Orbit in Toroidal B-Field



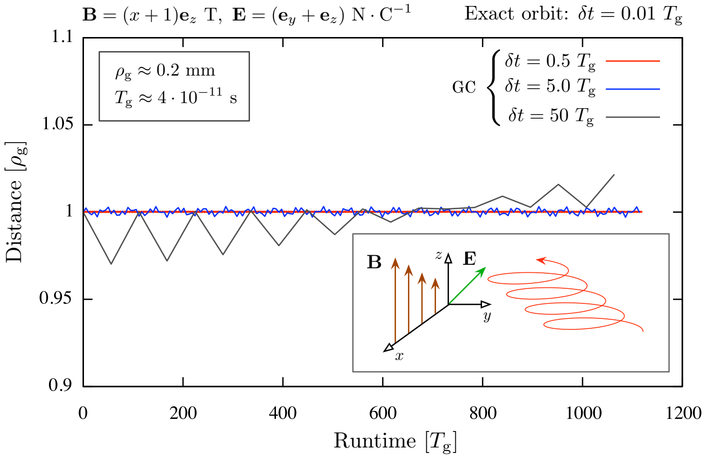
BS, $\delta t = 0.01\ T_g$

GC, $\delta t = 1\ T_g$

$$T_g \approx 4 \times 10^{-10}\ \text{s}$$
$$\rho_g \approx 2\ \text{mm}$$

Accuracy of Guiding Center Orbits

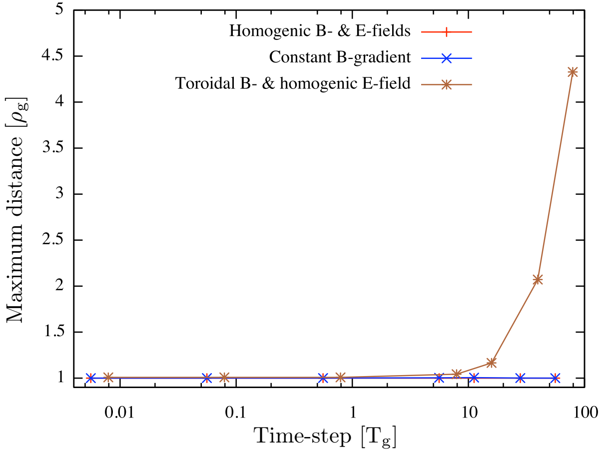
Distance Between Calculated Guiding Center & Electron Orbit over Runtime



Spatial Convergence Tests

Convergence of Guiding Center to Electron Orbit with Varying Time-Steps

$\rho_g \approx 0.2$ mm, $T_g \approx 4 \cdot 10^{-11}$ s



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Part III: External **B**-Fields

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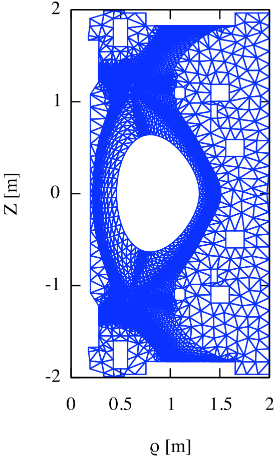
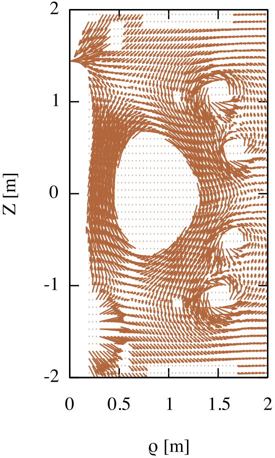
Given Field-Data

- External **B**-field.
- TOKAMAK $\rightarrow$ cylindrical symmetry.
- External **B**-field given on triangular mesh at $\varphi = 0$
  $\rightarrow$ given as constant on each triangle.
- Need for $\nabla\mathbf{B}$ $\rightarrow$ local interpolation.
- Requirement: $\nabla \cdot \mathbf{B} \stackrel{!}{=} 0$!

Triangle Mesh & Predefined B-Field

Projection on $\varphi = 0$ plane





Mesh Transformation

- Need for pointwise field definition → Transformation of triangular mesh.
- Original triangle → 3 new triangles.
- New fields defined on triangle vertices.
- 3-point interpolation within each triangle.

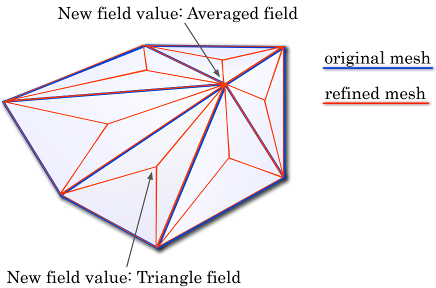


Figure: Mesh refinement & definition of new field values

Divergence Free Interpolation

Cylindrical symmetry:

- Divergence-freeness simplifies to:

$$\frac{\partial}{\partial\rho}(\rho B^{\rho}) + \frac{\partial}{\partial z}(\rho B^{z}) \overset{!}{=} 0 \tag{14}$$

- φ-component can be interpolated arbitrarily (e.g. linear).

$\longrightarrow$ Problem reduced to 2D-case: $\mathbf{F} := (\rho B^{\rho}, \rho B^{z})$.

2D-formulation:

Given: 3 *base-points* $\mathbf{r}_i \in \mathbb{R}^2$ and *base-values* $\mathbf{F}_i \in \mathbb{R}^2$, $i = 1, 2, 3$.

Sought: Field $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ with $\mathbf{F}(\mathbf{r}_i) = \mathbf{F}_i$ & $\nabla \cdot \mathbf{F} = 0$.

Ansatz: Component Separation

- General Ansatz for divergence-free field:

$$\begin{pmatrix} F^x \\ F^y \end{pmatrix} = \begin{pmatrix} F^x(y) \\ F^y(x) \end{pmatrix} \tag{15}$$

- Idea:
  - X component interpolated over Y coordinates of base-points.
  - Y component interpolated over X coordinates of base-points.

  $\longrightarrow$ simple linear interpolation for each component.
- Prerequisite: Interpolation base-points differ in both coordinates.
- Always possible in some coordinate system!
- Resulting **B**-field: differentiable almost everywhere!

Elaboration on Component Separation

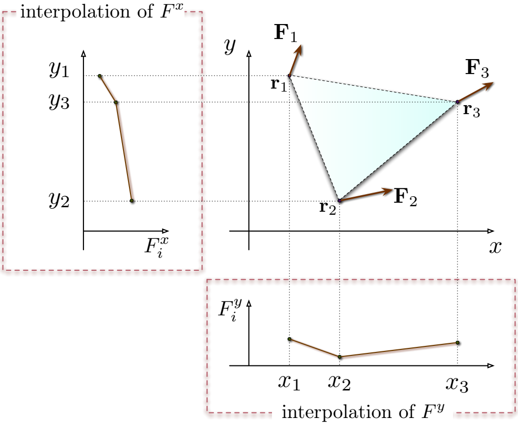


Figure: On component separated interpolation: Each component is defined by 1D-interpolation.

Example Interpolation

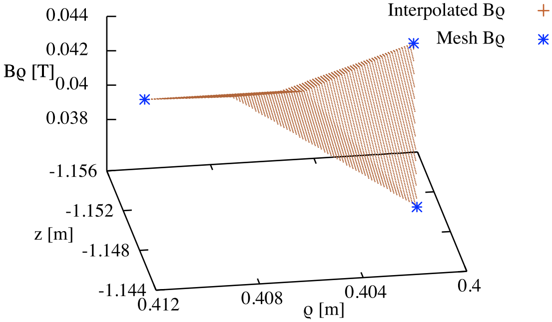


Figure: Example component-separated interpolation of B^{ρ} within 3 points.

Summary

- Charged particle orbits in **B**-fields characterized by high-frequency gyrations.
- **Problem:** Possible time-steps limited by electron gyration period.
- **Solution:** Exact particle orbit → replaced with guiding center orbit.
- **Implementation:** Allows time-step increase of up to $\times 100$.
- **B**-field given by divergence-free interpolation.

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Simulation of Drift-Kinetic Dynamics of Charged Particles in Magnetized Plasmas
Part V: Appendix

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Table of Contents

Magnetic Drift

- Caused by variation of gyration radius due to **B**-field variation.
- Always perpendicular to **B**-field.

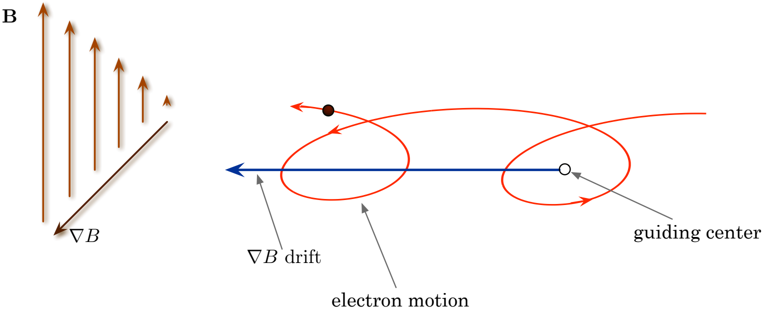

Figure: Magnetic drift of electron guiding center.

Acceleration Drift

- Caused by time-varying **B** & **E**-fields and field line curvature.
- In case of stationary fields: Perpendicular to **B**-field lines & curvature center.

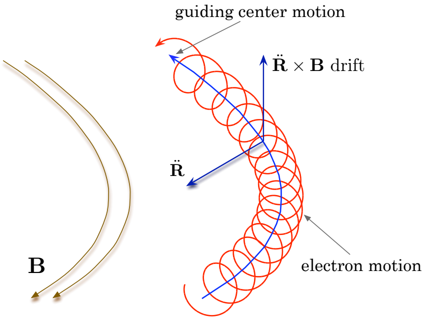

Figure: Acceleration drift of electron guiding center.

Guiding Center: μ-Convergence Tests