Stability of Clouds
and the Jeans Criterion

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May 12, 2010

Contents

- Introduction to molecular clouds.
- The Jeans stability criterium for molecular clouds.
- The process of collapse.

Molecular Clouds

Characterization

- Interstellar aggregations of matter.
- Consist mainly of molecules $(\mathrm{H_2}, \mathrm{CO})$: 99%.
- Typical temperatures $T \sim 10$ K.
- Typical densities $\rho \sim 10^{-22}\rho_{\odot}$.

Giant Molecular Clouds (GMC)

- Diameters up to 10^2 pc.
- Masses up to $10^6 M_{\odot}$.

The Cepheus B GMC

Figure: Composite image of Cepheus B, a GMC about 2400 Lj away. Red, green & blue data is in infrared, violet in X-Ray spectrum. The image span is about 3 pc. [NASA]

Star formation from GMCs

Importance of GMCs:

- Main source of star formation $\rightarrow$ "Stellar nurseries".
- Collapsing GMCs create newborn stars.
- Lifetime of a GMC $\approx 10^7$ j $\approx$ free fall time.

Collapse may be triggered by:

- Inner gravitational forces & instabilities.
- Shock-waves from nearby supernovae.
- Radiation pressure of nearby, massive stars.

Stability of GMCs

Forces acting on particles:

- Self-gravitational
- Internal gas pressure
- Magnetic fields, Radiation, Centrifugal, ..

Question: When does self-gravity lead to a collapse?

Depends on:

- Temperature
- Size
- Density
- Fields, Rotation, ..

Linear perturbation theory

Model:

- Homogeneous, isotropic background.
- Inviscid fluid.
- Only internal gravity and gas pressure.
- Background velocity $\mathbf{v}_0 = 0$.
- Constant background density ρ_0 and pressure p_0.
- Constant temperature $T \longrightarrow$ isothermal processes.

Sought: Evolution of perturbations.

Small density perturbations $\delta\rho \longrightarrow$ Linearization.

Linear perturbation theory

Equations:

- Euler equations of motion:

$$\frac{\partial \mathbf{v}}{\partial t} + (\nabla \mathbf{v}) \cdot \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \Phi \quad . \tag{1}$$

- Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad . \tag{2}$$

- Newton's law:

$$\Delta \Phi = 4\pi G \rho \quad . \tag{3}$$

- Equation of state: $p = p(\rho, T)$.

With: gravitational potential Φ, gravitational constant G, temperature T, pressure p, density ρ, velocity $\mathbf{v}$.

Linear perturbation theory

Starting point: Background solves equations, so should distorted fluid.

Linearization yields:

$$\frac{\partial^2 \delta\rho}{\partial t^2} = c_s^2 \cdot \Delta\delta\rho + 4\pi G\rho_0 \cdot \delta\rho \tag{4}$$

with isothermal speed of sound c_s.

Plane wave ansatz $\delta\rho \sim \exp[i(\mathbf{kx} - \omega t)]$ gives dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 - 4\pi G\rho_0 \quad . \tag{5}$$

$\rightarrow$ For large wavelengths λ, ω becomes imaginary!

Linear perturbation theory

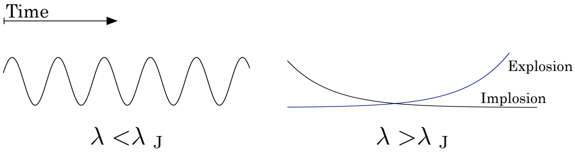
Conclusion:

- *Jeans length*

$$\boxed{\lambda_J := c_s \cdot \sqrt{\frac{\pi}{G\rho_0}}} \qquad (6)$$

is **maximum** wavelength for **stable** perturbation.

- Perturbations/inhomogeneities with scales $\gg \lambda_\mathrm{J}$ result in implosions/explosions.
- $\rightarrow \lambda_\mathrm{J}$ is estimation for maximal stable mass aggregations.



Alternative interpretation of the Jeans length

Consider cloud with density ρ_0:

- Notice:

$$\lambda_\mathrm{J} = (\text{Speed of sound}) \cdot (\text{Cloud free fall time}) \quad . \tag{7}$$

  $\rightarrow$ λ_J is distance a shock wave travels during free fall time of cloud.
- For diameters $D \ll \lambda_\mathrm{J}$: Shock waves traverse cloud faster than free fall time.
  $\rightarrow$ Rapid compensation of perturbations through gas pressure.
- For $D \gg \lambda_\mathrm{J}$: Cloud collapses faster than shock waves can cross it.
  $\rightarrow$ Gas pressure unable to compensate any perturbations.
  $\rightarrow$ cloud collapses.

Alternative interpretation of the Jeans length

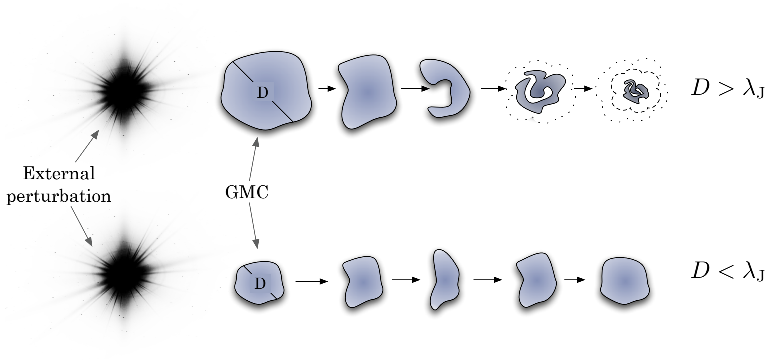


Figure: On the interpretation of the Jeans length, as maximal stable cloud diameter.

Example values

Assumptions

Gas	Ideal, Molecular Hydrogen
Density	$10^{-23}\rho_{\odot} \approx 10^{-20}\ \mathrm{Kg}\cdot\mathrm{m}^{-3}$
Temperature	$10\ \mathrm{K}$

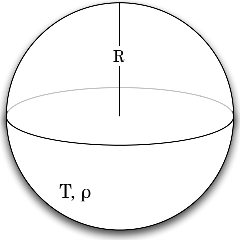
Yields

Isothermal speed of sound	$c_s \approx 50\ \mathrm{m}\cdot\mathrm{s}^{-1}$
Jeans length	$\lambda_J \approx 1 \times 10^{17}\ \mathrm{m} \approx 3\ \mathrm{pc}.$
	$\rightarrow$ approximately size of large molecular clouds.

The Virial theorem for spherical clouds

Model:

- Spherical, static mass aggregation of radius R.
- Constant density ρ & temperature T.
- Only force acting on particles: Gravitational.
- Particles have no internal degrees of freedom.



The Virial theorem for spherical clouds

Virial Theorem:

- Connects average potential & kinetic energy of system:

$$\langle E_{\mathrm{kin}} \rangle = -\frac{1}{2} \langle E_{\mathrm{pot}} \rangle \quad . \tag{8}$$

- Average kinetic energy = Thermal energy:

$$\langle E_{\mathrm{kin}} \rangle \approx \frac{3}{2} kT \cdot \frac{M}{\mu} \tag{9}$$

  with system mass M and particle mass μ.

- Potential energy = internal gravitational energy:

$$\langle E_{\mathrm{pot}} \rangle \approx -\frac{GM^2}{R} \quad . \tag{10}$$

The Virial theorem for spherical clouds

Leads to:

- Stability condition: $R \overset{!}{\approx} R_{\mathrm{J}}$, with stable radius

$$R_{\mathrm{J}} := \sqrt{\frac{kT}{\mu G \rho}} \quad . \tag{11}$$

  For greater radii, $E_{\mathrm{pot}} > E_{\mathrm{kin}}$ and gravity outweighs internal pressure.

- Note:

$$2R_{\mathrm{J}} = \frac{2c_s}{\sqrt{G\rho}} \approx \lambda_{\mathrm{J}} \quad . \tag{12}$$

  $\rightarrow$ Verification of the Jeans length as maximal, stable cloud diameter!

- Corresponds to maximum cloud mass

$$M_{\mathrm{J}} \approx \sqrt{\frac{k^3 T^3}{\mu^3 G^3 \rho}} \quad \text{(Jeans mass)} \quad . \tag{13}$$

Collapse of clouds: Isothermal stage of collapse

- Initially: Cloud density low.
  $\longrightarrow$ Radiation escapes easily.
  $\longrightarrow$ Isothermal process.
- As density increases, M_{J} decreases $\sim \rho^{-\frac{1}{2}}$.
  $\longrightarrow$ Collapse is accelerated even further.
  $\longrightarrow$ Approaches free fall!
- As M_{J} decreases, subregions become unstable and collapse.
  $\longrightarrow$ Fragmentation.

Collapse of clouds: Fragmentation

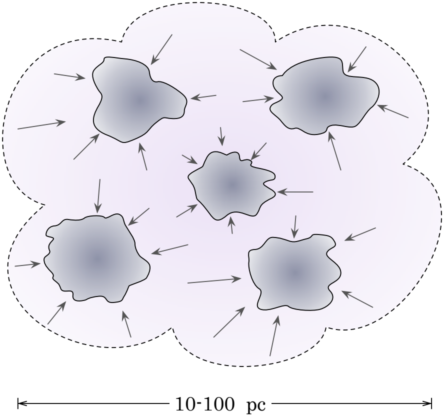


Figure: Fragmentation during the collapse of a GMC.

Collapse of clouds: Adiabatic stage of collapse

- As density increases $\longrightarrow$ Cloud becomes opaque to radiation.
- Process becomes adiabatic $\longrightarrow$ Temperature rises: $T \sim \rho^{\frac{2}{3}}$.
- Jeans mass rises: $M_{\mathrm{J}} \sim \rho^{\frac{1}{2}}$ $\longrightarrow$ Fragmentation halts.
- Collapse slows down $\longrightarrow$ Thermal time scale.

Estimating the final fragment size

During isothermal collapse:

- Energy scale $\sim E_{\mathrm{pot}} \sim GM^2/R$ and time scale $\sim (G\rho)^{-\frac{1}{2}}$ (free fall).
  $\rightarrow$ estimation for rate of energy release.
- Radiation rate limited by black body radiation at temperature T.

When collapse becomes adiabatic:

- Radiation has reached black body limit.
- Fragments have reach mass limit: $\sim M_J$.

Estimating the final fragment size

Results:

- Estimation for final fragment size:

$$M_{\mathrm{J}}^{\mathrm{Final}} \approx 0.025 \cdot M_{\odot} \cdot T^{\frac{1}{4}}\ \mathrm{K}^{-1} \tag{14}$$

(for atomic hydrogen).

- Setting $T \approx 10^3\ \mathrm{K}$, yields $M_{\mathrm{J}}^{\mathrm{Final}} \approx 0.2\ M_{\odot}$.
- Explains mass scales of typical stars!

Example simulation

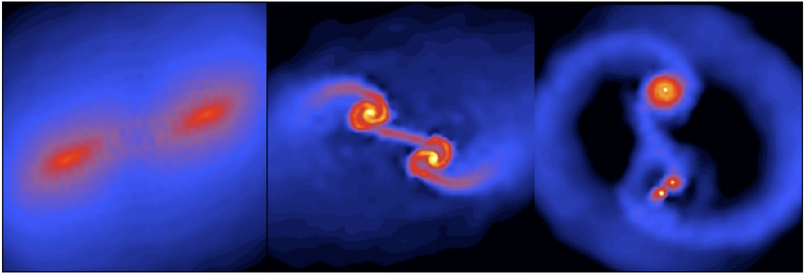


Figure: Example simulation of a collapsing molecular cloud, resulting in the formation of 3 new stars. Initial cloud temperature and span was 10 K and 0.01 pc respectively.[Bate et al]

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