Analysis III
FSU Jena - WS 07/08
Serie 01 - Lösungen

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Aufgabe 01

a)

$$G = \left\{(x,y) : x \in (-2,2),\ y \in \left(\frac{|x|}{2}, 1\right)\right\} = \{(x,y) : y \in (0,1),\ x \in (-2y, 2y)\}$$

b)

$$G = \left\{(x,y) : x \in \left(-\frac{1}{2}, \frac{1}{2}\right),\ y \in \left(\frac{1}{2} - \sqrt{\frac{1}{4} - x^2},\ \frac{1}{2} + \sqrt{\frac{1}{4} - x^2}\right)\right\}$$

Aufgabe 02

a)

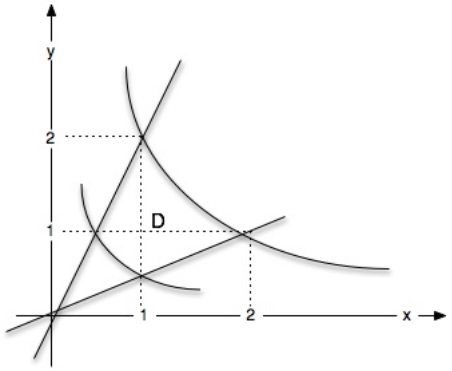
$$I_1 = \int_0^1 \left(\int_{1+\sqrt{1-y^2}}^{y+2} f(x,y)\ dx \right) dy$$

b)

$$I_2 = \int_0^a \left(\int_{\frac{y^2}{2a}}^{a-\sqrt{a^2-y^2}} f(x,y)\ dx + \int_{a+\sqrt{a^2-y^2}}^{2a} f(x,y)\ dx \right) dy + \int_a^{2a} \left(\int_{\frac{y^2}{2a}}^{2a} f(x,y)\ dx \right)\ dy$$

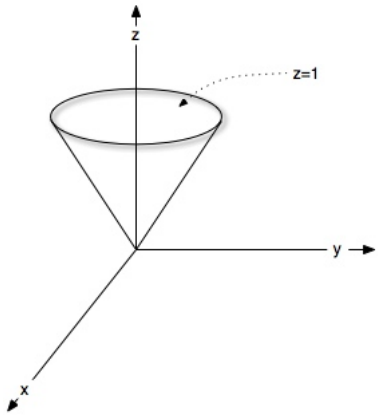
Aufgabe 03

Die Integration erfolgt über das unten abgebildete Gebiet



$$I_1 := \int\int_D y^2 d(x,y) = \int_{\frac{1}{2}}^1 \left(\int_{\frac{1}{2x}}^{2x} y^2\ dy \right) dx + \int_1^2 \left(\int_{\frac{x}{2}}^{\frac{2}{x}} y^2\ dy \right) dx = \int_{\frac{1}{2}}^1 \left(\frac{8x^3}{3} - \frac{1}{24x^3} \right) dx + \int_1^2 \left(\frac{8}{3x^3} - \frac{x^3}{24} \right) dx = \frac{45}{32}$$

Aufgabe 04



$$I = \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1} f(x,y,z)\ dz\ dx\ dy$$

$$I = \int_{0}^{1} \int_{-z}^{z} \int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x,y,z)\ dx\ dy\ dz$$

$$I = \int_{0}^{1} \int_{-z}^{z} \int_{-\sqrt{z^2-x^2}}^{\sqrt{z^2-x^2}} f(x,y,z)\ dy\ dx\ dz$$

$$I = \int_{-1}^{1} \int_{|y|}^{1} \int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x,y,z)\ dx\ dz\ dy$$

$$I = \int_{-1}^{1} \int_{|x|}^{1} \int_{-\sqrt{z^2-x^2}}^{\sqrt{z^2-x^2}} f(x,y,z)\ dy\ dz\ dx$$

Aufgabe 05

a)

$$I = \int_{0}^{1} \int_{y}^{1} \int_{0}^{xy} \left(xy^2z^3\right)\ dz\ dx\ dy = \frac{1}{364}$$

b)

$$I = \int_{0}^{1} \int_{0}^{x} \int_{0}^{xy} \left(xy^2z^3\right)\ dz\ dy\ dx = \frac{1}{364}$$

c)

$$I = \int_{0}^{1} \int_{\sqrt{z}}^{1} \int_{\frac{z}{x}}^{x} \left(xy^2z^3\right)\ dy\ dx\ dz = \frac{1}{364}$$